A Quadratic Static Game Model for Assessing the Impact of Climate Change

Bouchra Mroué[1(✉)], Anthony Couthures[1], Samson Lasaulce[1,2], and Irinel Constantin Morărescu[1,3]

[1] Université de Lorraine, CNRS, CRAN, 54000 Nancy, France
bouchra.mroue@univ-lorraine.fr
[2] Khalifa University, Abu Dhabi, UAE
[3] Technical University of Cluj-Napoca, Cluj-Napoca, Romania

Abstract. The Conference of Parties (COP) aims to address the global warming problem through agreements for reducing emissions. However, the current strategies fall short due to minimal efforts for emission reductions driven by short-term economic considerations. To better understand under which circumstances countries may have more virtuous behavior, we propose a static game model. In this model, the players are represented by governments, and their actions correspond to emissions levels. The utilities are a trade-off between economic benefits and damage caused by climate change. A key feature of the game is that it is parameterized by a state, which is precisely the state of the climate dynamics. We conduct the Nash equilibrium analysis. In the numerical analysis, we assess the impact of the damage function on the behavior of the governments.

Keywords: Game theory · Climate change models · Complex systems · Static game · Potential game

1 Introduction

Global warming is a major environmental concern. The Conference of Parties (COP) was created to provide solutions to it. Nevertheless, one can see that efforts to emit fewer CO_2 are not very significant. On the contrary, the total of carbon emissions keeps on increasing whereas the effects of climate change have been made more apparent over the last decades. The objective of this paper is to provide some insights into the complex decision-making process of networked countries that optimize their utility functions by taking into account the dynamics of the global atmospheric temperature and CO_2 concentration. Doing so we provide elements that explain why the CO_2 emissions do not drastically reduce and exhibit some modelling conditions under which CO_2 reductions indeed occur.

The state of the art on this subject contains both geophysical-type and economic-type studies mainly based on empirical or ad-hoc strategies [15,16].

© The Author(s), under exclusive license to Springer Nature Switzerland AG 2024
H. Cherifi et al. (Eds.): COMPLEX NETWORKS 2023, SCI 1142, pp. 383–394, 2024.
https://doi.org/10.1007/978-3-031-53499-7_31

Typical formal economic analyses do not integrate the geophysical aspect of the problem (see [1,3] where the temperature dynamics are ignored). A neat game formulation that includes examining coalition formation, financial transfers, and cost-sharing was presented in [17]. This work provides insights into the complexity of environmental cooperation, coalition stability, and the design of efficient and stable agreements. Still, [17] only provides a game-theoretic analysis without considering the temperature but only CO_2 concentration dynamics. We also note that most of the existing game-theoretic studies are based on the work of Finus and his co-authors ([5–7]).

While the literature on climate change is quite rich, to our knowledge, no formal game-theoretic work has been conducted where both geophysical aspects and strategic aspects are considered and modeled mathematically. Models which couple economic aspects and climate science are referred to as integrated assessment models (IAMs). Among the most famous IAMs one can find the DICE model introduced by the Nobel Prize winner W. Nordhaus and his collaborators [14]. For convenience, they usually use simple climate model which matches the elaborate and complex geophysical models used by the IPCC (Intergovernmental Panel on Climate Change) [9].

In this study, we make the following key contributions to the field of environmental game theory:

- We propose a novel static game that is potential and where the utility function is a function of the geophysical state and the players' actions.
- We provide the expression of the unique pure Nash equilibrium, in some sufficient conditions, for the quadratic case functions.
- We assess numerically, the effects of the economic damage function due to climate change modeling on the behavior of the countries in terms of CO_2 emissions.

The rest of the paper is organized as follows. Section 2 is dedicated to the presentation of a simple but well-established climate model and its ingredients. The problem analyzed in this work is formulated and the subsequent game-theoretical analysis is provided in Sect. 3, where we study the existence and the uniqueness of the Nash equilibrium in a specific case. Numerical simulations illustrate our results in Sect. 4 and provide several insights or societal interest. We conclude the paper and give some perspectives in Sect. 5.

2 Simple Climate Model (SCM)

Let us first present the different pieces of a simple climate model that are coupled with the game introduced further in the paper. In our game, we are using the climate structure of IAMs involving three key ingredients: the carbon cycle (CC), the radiative forcing (RF), and the temperature dynamics (TD).

2.1 Carbon Cycle Model

In the sequel, we are using the CC model employed in [10] referred to as Joos model. This model was designed to fit the impulse response functions to a set of Earth System model simulations done by the CMIP5 model of IPCC. Denoting by C the vector of CO_2 concentrations in some boxes, where a box represents, in most of the carbon models, the proportion of carbon decaying with respect to the half-life of carbon in these virtual boxes. We get the following linear model:

$$\begin{aligned} C(t+1) &= \mathbf{A}_C C(t) + b_C E(t), \\ C_{\mathrm{AT}}(t) &= \ d_C^{\top} C(t). \end{aligned} \tag{1}$$

where

$$\mathbf{A}_C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.9975 & 0 & 0 \\ 0 & 0 & 0.9730 & 0 \\ 0 & 0 & 0 & 0.7927 \end{bmatrix}, \quad b_C = \begin{bmatrix} 0.2173 \\ 0.2240 \\ 0.2824 \\ 0.2763 \end{bmatrix}, \quad \text{and} \quad d_C = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

2.2 Radiative Forcing

The RF represents the impact of greenhouse gas (GHG) accumulation on the global radiation balance. The climate equation calculates the average surface temperature of the earth and the average deep-sea temperature at each time step. The relationship between the accumulation of GHGs and the increase in RF is derived from empirical measurements. A mathematical representation is given as:

$$F(t) = F_{2\times \mathrm{CO}_2} \log_2 \left(\frac{C_{\mathrm{AT}}(t)}{C_{\mathrm{AT,ref}}} \right) + F_{\mathrm{nonCO}_2}(t), \quad \forall t \in \mathbb{R}^+, \tag{2}$$

where $C_{\mathrm{AT,ref}}$ is the CO_2 concentration in 1750 that is considered as a reference since it is the pre-industrial time equilibrium, $F_{2\times \mathrm{CO}_2}$ is a parameter fitted from data and $F_{\mathrm{nonCO}_2}(t)$ is the radiative forcing caused by other GHGs. In some models, such as FUND and PAGE, $F_{\mathrm{nonCO}_2}(t)$ is modeled by the dynamics of methane and nitrous oxide while for DICE it represents exogenous forcing. In this work we use the exogenous forcings proposed by [11], given by:

$$F_{\mathrm{nonCO}_2}(t) = f_0 + \min \left\{ f_1 - f_0, \frac{f_1 - f_0}{t_f}(t-1) \right\}, \quad \forall t \in \mathbb{R}^+, \tag{3}$$

where f_0 and f_1 are respectively the forcing of GHGs other than CO_2 in 2010 and in 2100 and t_f is the time step.

2.3 Temperature Dynamic Model

The final part of an SCM is the Temperature Dynamics model. This allows us to describe the evolution of the global atmospheric temperature based on a specific

radiative forcing. The literature on the modeling of TD is very rich and we focus on the model proposed in [8] referred to as Geoffroy model:

$$\begin{cases} \theta(t+1) = \mathbf{A}_\theta \theta(t) + b_\theta F(t+1), \\ \theta_{\mathrm{AT}}(t) = \quad d_\theta^\top \theta(t). \end{cases} \tag{4}$$

where

$$\mathbf{A}_\theta = \begin{bmatrix} 1-(\lambda+\mu)/c & \mu/c \\ \mu/c_0 & 1-\mu/c_0 \end{bmatrix}, \quad b_\theta = \begin{bmatrix} 1/c \\ 0 \end{bmatrix}, \quad \text{and} \quad d_\theta = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{5}$$

We highlight that the dynamics depend on some constants: c as the effective heat capacity of the upper/mixed ocean layer, c_0 as the effective heat capacity of the oceans, λ and μ are chosen with all these constants such that they best fit the multi-model mean of the CMIP5 set which is the model used by IPCC [4]. In the sequel, we use the following values: $c = 7.3$, $c_0 = 106$, $\lambda = 1.13$, and $\mu = 0.73$. Denoting the state of the system by $x = (\theta, C)$, the atmospheric CO_2 concentration and the atmospheric temperature can be described as follows:

$$C_{\mathrm{AT}}(t+1) = C_{\mathrm{AT}}(x(t), a(t)) = \psi_C(x(t)) + \tilde{b}_C \sum_{n=1}^{N} a_n(t), \tag{6a}$$

$$\theta_{\mathrm{AT}}(t+1) = \theta_{\mathrm{AT}}(x(t), a(t)) = \psi_\theta(x(t)) + \tilde{b}_\theta \ln\left(\psi_C(x(t)) + \tilde{b}_C \sum_{n=1}^{N} a_n(t)\right), \tag{6b}$$

where $\tilde{b}_C = d_C^\top b_C$, $\tilde{b}_\theta = d_\theta^\top b_\theta F_{2\times \mathrm{CO}_2} / \ln 2$ are positive parameters and ψ_C, ψ_θ are functions of the state defined as $\psi_C(x(t)) = d_C^\top \mathbf{A}_C C(t)$ and

$$\psi_\theta(x(t)) = d_\theta^\top \mathbf{A}_\theta \theta(t) + d_\theta^\top F_{\mathrm{nonCO_2}}(t) - d_\theta^\top b_\theta F_{2\times \mathrm{CO}_2} \log_2 C_{\mathrm{AT,ref}}.$$

3 Game-Theoretic Analysis

3.1 Carbon Emission Game Model

This paper considers a static climate game Γ over a set of players $\mathcal{N} = \{1, \ldots, N\}$ that represent non-identical countries. The goal of each player is to maximize their utility which is a trade-off between their benefits as functions of their emissions and a weighted global damage as a function of the global atmospheric temperature. The player's action is the CO_2 emissions that they are planning to emit over the time step between two decisions (COP meetings). We note that $e_n^{\min}$ and $e_n^{\max}$ are respectively the minimum and the maximum emissions that player n can emit. The action set is $\mathcal{A} = \prod_{n\in\mathcal{N}} \mathcal{A}_n$ where $\mathcal{A}_n = [e_n^{\min}, e_n^{\max}] \subset \mathbb{R}$ is the set of actions for the player $n \in \mathcal{N}$. We shall also use a_{-n} to denote the vector of the actions of all the players except player n. The corresponding set of actions is $\mathcal{A}_{-n}$. We denote by $x \in \mathbb{R}^6$ the state of the system, by $a_n \in \mathcal{A}_n$ the action of player n, and by $a \in \mathcal{A}$ the vector of all actions. Note that the atmospheric temperature is a function of the state and of the action i.e., $\theta_{\mathrm{AT}}(x, a)$.

Definition 1. *The utility function for the n^{th} player is chosen to be a difference between an individual benefit function B_n and a (weighted) global cost/damage function D:*

$$u_n(x,a) = \sum_{i=0}^{2} \beta_{i,n} a_n^i - w_n \sum_{i=0}^{2} \gamma_i \theta_{\mathrm{AT}}^i(x,a) := B_n(a_n) - w_n D\left(\theta_{\mathrm{AT}}(x,a)\right), \quad (7)$$

where w_n is a positive weight that measures the economic impact of climate change on player n. In the sequel we denote by $(u_n)_{n\in\mathcal{N}}$, the family of utility functions which defines the strategic form of the static game under study.

3.2 Existence and Uniqueness of a Pure Nash Equilibrium

A key solution concept for the interactive situation which involves several players, each aiming to maximize its own utility function, is given by the Nash equilibrium. A Nash equilibrium can be interpreted as a possible forecast for such a situation where decisions are interdependent as they are for the global carbon emission problem. An important property for a game is precisely to know whether it possesses a pure Nash equilibrium. It turns out that, by construction, the game under study has always a pure Nash equilibrium. This is because it belongs to the class of weighted potential games as defined by Monderer and Shapley [12].

A game $\Gamma = (\mathcal{N}, \mathcal{A}, \mathcal{U})$ is a weighted potential game if and only if there exists a potential function $\phi : \mathcal{A} \mapsto \mathbb{R}$ and $(\delta_n)_{n\in\mathcal{N}}$ a vector of positive weights, such that, for all $n \in \mathcal{N}, a_n, \tilde{a}_n \in \mathcal{A}_n; a_n \neq \tilde{a}_n$ and $a_{-n} \in \mathcal{A}_{-n}$ one has $u_n(a_n, a_{-n}) - u_n(\tilde{a}_n, a_{-n}) = \delta_n \left[\phi(a_n, a_{-n}) - \phi(\tilde{a}_n, a_{-n})\right]$.

It can be checked that the following function ϕ is a potential for the considered game with weights $(w_n)_{n\in\mathcal{N}}$:

$$\phi\left(x,a\right) = \sum_{n=1}^{N} \frac{1}{w_n} \sum_{i=0}^{2} \beta_{i,n} a_n^i - \sum_{i=0}^{2} \gamma_i \theta_{\mathrm{AT}}^i(x,a). \quad (8)$$

The previous results yield the existence of at least one pure Nash equilibrium for the quadratic case. Next, we will provide a necessary condition for uniqueness. Actually, we consider a concave benefit function and a convex damage function, since the connection between GDP and emissions is frequently represented through a concave function. Also, a quadratic damage convex function is mostly used in the economic literature focusing on the consequences of climate change.

Proposition 1. *Supposing that $\gamma_2 > 0$ and for all $n \in \mathcal{N}$, if*

$$\frac{\gamma_2 \tilde{b}_\theta^2 \tilde{b}_C w_n}{e_n^{\max}} \exp\left(\frac{\gamma_1 + 2\gamma_2 \psi_\theta(x)}{2\gamma_2 \tilde{b}_\theta} - 1\right) - \frac{\beta_{1,n}}{2e_n^{\max}} < \beta_{2,n} < 0. \quad (9)$$

Then the pure Nash equilibrium is unique and it corresponds to all players emitting to the maximum, i.e., $a_n = e_n^{\max}$, $\forall n \in \mathcal{N}$.

Proof. If $\min_{a_n \in \mathcal{A}_n} B'_n(a_n)/w_n > \max_{a_n \in \mathcal{A}_n} \partial \left[D\left(\theta_{\mathrm{AT}}\left(x,a\right)\right)\right]/\partial a_n$, then one has a unique pure Nash equilibrium. Indeed, in this case, the utility functions will be strictly increasing and then the maximum is attained when all players emit the maximum of possible emissions. Straightforward computation shows that:

$$\min_{a_n \in \mathcal{A}_n} B'_n(a_n) = \min_{a_n \in \mathcal{A}_n} \left[\beta_{1,n} + 2\beta_{2,n} a_n\right] \overset{\text{if } \beta_{2,n}<0}{=} \beta_{1,n} + 2\beta_{2,n} e_n^{\max}. \tag{10}$$

On the other hand:

$$\frac{\partial \left[D\left(\theta_{\mathrm{AT}}\left(x,a\right)\right)\right]}{\partial a_n} = \frac{\tilde{b}_\theta \tilde{b}_C \left[\gamma_1 + 2\gamma_2 \left[\psi_\theta(x) + \tilde{b}_\theta \ln\left(\psi_C(x) + \tilde{b}_C \sum_{n=1}^N a_n\right)\right]\right]}{\psi_C(x) + \tilde{b}_C \sum_{n=1}^N a_n}. \tag{11}$$

In this part of the proof, we will use the parameters $a, b, c, d, k \in \mathbb{R}$ that are constants, and independent of the problem formulated before. They are used to ease the presentation of the variation of the function. To find the maximum of (11), we consider the case where $\gamma_2 > 0$. Let us find the maximum of the function $f : \mathbb{R} \to \mathbb{R}$ for $a, b, c, k \in \mathbb{R}_+^*, d \in \mathbb{R}$, given for all $z \in \mathbb{R}$ by:

$$f(z) = \frac{k(d + c\ln\left(a + bz\right))}{a + bz}.$$

When differentiating f with respect to $z \in \mathbb{R}$, we find the unique root of f' given by $z_0 = \left(e^{(1-d/c)} - a\right)/b$. Then simply computing $f'\left((e^{-d/c} - a)/b\right) = kbc/e^{(4-2d/c)} > 0$, and $f'\left((e^{(2-d/c)} - a)/b\right) = -kbc/e^{(4-2d/c)} < 0$ provides that f' is strictly decreasing. We can conclude that the function f is strictly concave and reaches its maximum at z_0, given by $f(z_0) = kc\exp\left(d/c - 1\right)$. Now by using f with $a = \psi_C(x) + \tilde{b}_C \sum_{m=1, m\neq n}^N a_m$, $b = \tilde{b}_C$, $c = 2\gamma_2\tilde{b}_\theta$, $d = \gamma_1 + 2\gamma_2\psi_\theta(x)$ and $k = \tilde{b}_\theta\tilde{b}_C$, we conclude that the maximum of D' is given by:

$$\max_{a_n \in \mathcal{A}_n} \frac{\partial \left[D\left(\theta_{\mathrm{AT}}\left(x,a\right)\right)\right]}{\partial a_n} = 2\gamma_2 \tilde{b}_\theta^2 \tilde{b}_C \exp\left(\frac{\gamma_1 + 2\gamma_2\psi_\theta(x)}{2\gamma_2\tilde{b}_\theta} - 1\right).$$

After minimizing the benefit variations and maximizing the damage variations we get that $\min_{a_n \in \mathcal{A}_n} B'_n(a_n)/w_n > \max_{a_n \in \mathcal{A}_n} \partial \left[D\left(\theta_{\mathrm{AT}}\left(x,a\right)\right)\right]/\partial a_n$ is equivalent to:

$$\frac{\beta_{1,n} + 2\beta_{2,n} e_n^{\max}}{w_n} > 2\gamma_2 \tilde{b}_\theta^2 \tilde{b}_C \exp\left(\frac{\gamma_1 + 2\gamma_2\psi_\theta(x)}{2\gamma_2\tilde{b}_\theta} - 1\right),$$

which is equivalent to (9). □

Proposition 1 basically states that looking at the short term, all the countries will emit as much as possible as long as the damage function does not have a sufficiently large impact. This can be changed either by considering less optimistic damage functions or looking at the long-term behavior when the atmospheric temperature is higher which will lead to larger damages.

3.3 Expression of the Nash Equilibrium

In this section, the goal is to express the Nash equilibrium actions for the player. The motivation for this is twofold; it makes interpretations much easier (e.g., the impact of radiative forcing or the damage severity level on the behavior of the countries) and it renders the problem of computing the equilibrium very simple to solve. To express the NE, let us assume from now on that ϕ is strictly concave. The pure NE is denoted by $a^* = (a_1^*, \ldots, a_N^*)$ where either there exists $n \in \mathcal{N}$ such that $a_n^* \in \{e_n^{\min}, e_n^{\max}\}$, or a^* is an interior NE. In the later case, the players will tend to reduce their emissions.

We recall that the potential function is defined by (8) with the atmospheric temperature θ_{AT} given by:

$$\theta_{\mathrm{AT}}(x, a) = \psi_\theta(x) + \tilde{b}_\theta \ln\left(\psi_C(x) + \tilde{b}_C \sum_{n=1}^{N} a_n\right). \tag{12}$$

Proposition 2. *If ϕ is strictly concave and differentiable, the Nash is the vector $a^* = (a_1^*, \ldots, a_N^*)$ that satisfies, for all $n \in \mathcal{N}$, the following N equations*

$$\frac{1}{w_n} B_n'(a_n) - \frac{\partial\left[D\left(\theta_{\mathrm{AT}}(x, a)\right)\right]}{\partial a_n} = \overline{\lambda}_n - \underline{\lambda}_n. \tag{KKT}$$

with $\overline{\lambda}_n, \underline{\lambda}_n \geq 0$ with $\underline{\lambda}_n^*(a_n^* - e_n^{\min}) = 0$ and $\overline{\lambda}_n^*(a_n^* - e_n^{\max}) = 0$ being the KKT multipliers with associated constraints.

Proof. The proof is straightforward: ϕ is continuous over $\mathcal{A}$, then there exists a NE, a^*. Moreover, if ϕ is strictly concave, then the NE is unique. Since the constraints are linear we can apply the KKT conditions. □

In the proposition below, we provide sufficient conditions to express the NE. To do so, we will need the following lemma on the zeros of the equation $rs^2 + ps + q = k \ln(rs + v)$.

Lemma 1. *For $k, p, q \in \mathbb{R}$, $r, v \in \mathbb{R}_+^*$, and for all $s \in \mathbb{R}$ such that $rs + v > 0$, the following equation in s:*

$$rs^2 + ps + q = k \ln(rs + v),$$

- *has at most one solution if $rk + (2v - p)^2/8 < 0$,*
- *has at most two solutions if $rk + (2v - p)^2/8 = 0$,*
- *has at most three solutions if $rk + (2v - p)^2/8 > 0$.*

When ϕ is strictly concave, we can apply the KKT conditions to find the unique pure NE. Let us assume that ϕ is continuous on $\mathcal{A}$, so there exists a pure NE, denoted by a^*. Applying the Proposition 2, we have for every $n \in \mathcal{N}$, a^* verifies (KKT). This leads to the sufficient condition of the unique interior NE given in the following proposition.

Proposition 3. *Assuming that ϕ is strictly concave with $\beta_{2,n} \neq 0, \forall n \in \mathcal{N}$, and denoting $\Lambda_n := (w_n(\underline{\lambda}_n - \overline{\lambda}_n) - \beta_{1,n})/2\beta_{2,n}$. If*

$$\tilde{b}_\theta^2 \tilde{b}_C^2 \gamma_2 \sum_{n=1}^{N} \frac{w_n}{\beta_{2,n}} + \frac{\left(\psi_C(x) + \tilde{b}_C \sum_{n=1}^{N} \Lambda_n\right)^2}{8} < 0, \tag{13}$$

then the unique pure NE of the game Γ is given by $a^* = (a_1^*, \ldots, a_N^*)$, where for all $n \in \mathcal{N}$,

$$a_n^* = \Lambda_n + \frac{w_n \tilde{b}_\theta \tilde{b}_C \left[\gamma_1 + 2\gamma_2 \psi_\theta(x) + 2\gamma_2 \tilde{b}_\theta \ln\left(\psi_C(x) + \tilde{b}_C \tilde{S}\right)\right]}{2\beta_{2,n}\left(\psi_C(x) + \tilde{b}_C \tilde{S}\right)}, \tag{14}$$

where $\tilde{S}$ is the unique solution of the equation $rs^2 + ps + q = k \ln(rs + v)$, with $r = \tilde{b}_C$, $p = \psi_C(x) - \tilde{b}_C \sum_{n=1}^{N} \Lambda_n$, $k = \tilde{b}_\theta^2 \tilde{b}_C \gamma_2 \sum_{n=1}^{N} \beta_{2,n}/w_n$, $v = \psi_C(x)$, and $q = -\psi_C(x) \sum_{n=1}^{N} \Lambda_n - \sum_{n=1}^{N} w_n \tilde{b}_\theta \tilde{b}_C (\gamma_1 + 2\gamma_2 \psi_\theta)/2\beta_{2,n}$.

Proof. Using Proposition 2, for all $n \in \mathcal{N}$ one has

$$\frac{1}{w_n}(\beta_{1,n} + 2\beta_{2,n} a_n^*) - [\gamma_1 + 2\gamma_2 \theta_{\mathrm{AT}}(x, a^*)] \frac{\partial \theta_{\mathrm{AT}}}{\partial a_n}(x, a^*) = \underline{\lambda}_n - \overline{\lambda}_n.$$

Dividing by $\beta_{2,n} \neq 0$ and using the notation Λ_n introduced in the statement, we get that $\forall n \in \mathcal{N}$,

$$a_n^* = \Lambda_n + \frac{w_n \tilde{b}_\theta \tilde{b}_C \left[\gamma_1 + 2\gamma_2 \psi_\theta(x) + 2\gamma_2 \tilde{b}_\theta \ln\left(\psi_C(x) + \tilde{b}_C \sum_{n=1}^{N} a_n^*\right)\right]}{2\beta_{2,n}\left(\psi_C(x) + \tilde{b}_C \sum_{n=1}^{N} a_n^*\right)}. \tag{15}$$

Let us introduce the notation $s := \sum_{n=1}^{N} a_n^*$. Summing (15) over $n \in \mathcal{N}$ yields:

$$s = \sum_{n=1}^{N} \Lambda_n + \sum_{n=1}^{N} \frac{w_n}{2\beta_{2,n}} \frac{\tilde{b}_\theta \tilde{b}_C \left[\gamma_1 + 2\gamma_2 \psi_\theta(x) + 2\gamma_2 \tilde{b}_\theta \ln\left(\psi_C(x) + \tilde{b}_C s\right)\right]}{\psi_C(x) + \tilde{b}_C s},$$

which can be re-written as $rs^2 + ps + q = k \ln(rs + v)$ with r, p, q, v, and k given in the statement above. By using the results of the lemma 1, we conclude that if (13) is verified then there exists at most one solution $\tilde{S}$ of the equation $rs^2 + ps + q = k \ln(rs + v)$ in $\left[\sum_{n=1}^{N} e_n^{\min}, \sum_{n=1}^{N} e_n^{\max}\right]$. Thus, the NE is unique.

4 Numerical Analysis

We will illustrate the previous theoretical results and the behavior of the proposed model. We have implemented a code that allows us to play with the parameters, whether they are from the SCM or the benefit and damage functions. We set $N = 6$ with the parameters specified in Table 1, where AOC refers

Table 1. Specific values for each player in 2020.

Player	China	USA	EU	India	Russia	AOC
$e_n^{\max}$ ($GtCO_2/y$)	11	7	4	3	2	5
$GDP_n^{\max}$ ($10^9\$$)	14630	19290	13890	2500	1420	11640
w_n	1.1847	1.1941	1.1248	0.9074	1.2866	1.1847

to all other countries. For the presented graphs, we will use the CC from [10] and the TD from [8], as they are often considered the closest to the IPCC results [4,13].

We illustrate the case where the benefit function is quadratic in a_n, and the damages are quadratic and re-scaled, i.e.,

$$u_n(x,a) = \mathrm{GDP}_n^{\max}\left(2\frac{a_n}{e_n^{\max}} - \left(\frac{a_n}{e_n^{\max}}\right)^2 - w_n\left[D\left(\theta_{\mathrm{AT}}(x,a)\right)\right]^{\alpha}\right), \tag{16}$$

where α represents the power of the damages and measures the severity level of climate change on the economics. The static game is played repetitively every five years until 2100 while updating $e_n^{\max}$ and $\mathrm{GDP}_n^{\max}$ at each iteration of the game. For more information, refer to [2].

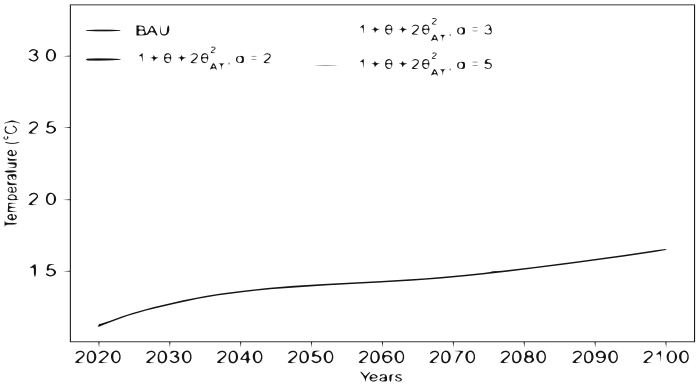


Fig. 1. The increase of the forecast temperature due to the CO_2emissions at NE in different scenarios.

It is interesting to note that higher α induces higher damages and consequently lower CO_2 emissions and smaller increases in the temperature. For large α (e.g., $\alpha = 5$), China, the USA, the EU, and AOC reduce their emissions until they completely stop emitting (see 2). In (Fig. 1), the temperatures in 2100 range from around $+3.2°C$ for low damages, resembling a Business-as-Usual (BAU) scenario, to $+1.6°C$ for high damages. These temperature levels are in line with

the projections of the IPCC [13] and correspond to the emission trajectories of the countries. In order to prevent the over-warming of the planet by 2100 we need to revise the modeling of the economic damages and change the strategies accordingly.

Table 2. Time at which the countries stop emitting versus α (which measures the economic damage due to climate change). The symbol / means no stopping.

Player	$\alpha = 1$	$\alpha = 2$	$\alpha = 3$	$\alpha = 4$	$\alpha = 5$	$\alpha = 6$	$\alpha = 7$	$\alpha = 8$	$\alpha = 9$	$\alpha = 10$
China	/	/	2065	2020	2020	2020	2020	2020	2020	2020
USA	/	/	2075	2020	2020	2020	2020	2020	2020	2020
EU	/	/	/	2045	2020	2020	2020	2020	2020	2020
India	/	/	/	/	/	/	/	2095	2085	2080
Russia	/	/	/	/	/	/	/	/	/	/
AOC	/	/	2080	2025	2020	2020	2020	2020	2020	2020

Table 2 shows that if the damages are not significant, i.e. α is small, the CO_2emissions of the players will not stop before 2100. Low damage hampers the cooperation recommended by the IPCC. When α is large enough, the NE strategies of the players are to stop emitting as soon as possible. Except for Russia which continues to emit no matter how big is the damage, and for India which stops emitting only when $\alpha \geq 8$. This can be explained by the fact that the benefits of India and Russia are still very big compared with the corresponding loss. The product $w_n \mathrm{GDP}_n^{\max}$ has to be increased for these countries in order to stop their emissions.

5 Conclusion

In this paper, we have introduced and analyzed a static game that provides some insights into the strategic behavior of governments in terms of carbon emissions with respect to climate change. Each government is assumed to implement a trade-off between a benefit due to emitting and damage due to climate change. The Nash equilibrium of the game being a suitable solution concept, we have studied the existence and uniqueness of the equilibrium and also addressed the problem of expression of the actions at equilibrium. The numerical analysis provides several insights into the carbon emission problem. For instance, it is seen that to reach the Paris Agreement on climate (namely, maintain the temperature excess below 2°C), the damage to climate change has to be significant enough. This constitutes a sufficient condition under which governments will spontaneously reduce their emissions. Depending on the severity level of the damage (which is measured by the exponent α), governments are incited to stop emitting CO_2 and it is shown to be possible to (roughly) forecast a time at which a country stops emitting. The obtained times are typically higher than values

claimed publicly (e.g., 2050). To conclude this paper, we would like to mention several extensions of the present work. First, the present problem formulation might be enriched by considering the planning aspect for which countries consider long-term utilities instead of short-term ones. Second, the emergence of cooperation might be studied by considering other solution concepts such as the social optimum or Nash bargaining solution. Thirdly, the damage functions might be more individualized while maintaining the potential structure of the game. At last, the present work can be seen as the first necessary step to be taken to study formally repeated interactions between the players, e.g., through a repeated or stochastic game model. In such a framework, it might be assumed that the action and state of a country are not perfectly observed by the others, which defines a non-trivial observation graph to be taken into account in the equilibrium analysis.

References

1. Al Khourdajie, A., Finus, M.: Measures to enhance the effectiveness of international climate agreements: the case of border carbon adjustments. Eur. Econ. Rev. **124**, 103405 (2020)
2. Anthony, C., Bouchra, M.: Co2-game (2023). https://doi.org/10.5281/zenodo.8283364
3. Bosetti, V., et al.: Incentives and stability of international climate coalitions: an integrated assessment. Energy Policy **55**, 44–56 (2013)
4. Dietz, S., et al.: Are economists getting climate dynamics right and does it matter. J. Assoc. Environ. Resour. Economists **8**, 895–921 (2021)
5. Finus, M.: Game theoretic research on the design of international environmental agreements: insights, critical remarks, and future challenges. Int. Rev. Environ. Resour. Econ. **2**, 29–67 (2008)
6. Finus, M.: Game Theory and International Environmental Cooperation: Any Practical Application? Edward Elgar Publishing (2000)
7. Finus, M., et al.: The efficacy of international environmental agreements when adaptation matters: Nash-Cournot vs Stackelberg leadership. J. Environ. Econ. Manage. **109**, 102461 (2021)
8. Geoffroy, O., et al.: Transient climate response in a two-layer energy-balance model. Part I: analytical solution and parameter calibration using CMIP5 AOGCM experiments. J. Clim. **26**, 1841–1857 (2013)
9. IPCC, Annex II: Models [Gutiérrez, J M., A.-M. Tréguier (eds.)], in: Masson-Delmotte, V., et al. (Eds.), Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change (2021)
10. Joos, F., et al.: Carbon dioxide and climate impulse response functions for the computation of greenhouse gas metrics: a multi-model analysis. Atmos. Chem. Phys. **13**, 2793–2825 (2013)
11. Kellett, C.M., et al.: Feedback, dynamics, and optimal control in climate economics. Annu. Rev. Control **47**, 7–20 (2019)
12. Monderer, D., Shapley, L.S.: Potential games. Games Econ. Behav. **14**, 124–143 (1996)

13. Mukherji, A., et al.: Synthesis report of the IPCC sixth assessment report (AR6) (2023)
14. Nordhaus, W., Sztorc, P.: DICE 2013R : Introduction and User's Manual (2013). URL https://sites.google.com/site/williamdnordhaus/dice-rice
15. Perissi, I., Jones, A.: Investigating EU decarbonization strategies: evaluating the pathway to carbon neutrality by 2050. Sustainability **14**, 4728 (2022)
16. Solano, B., et al.: Decarbonizing the EU energy system by 2050: an important role for BECCS. Clim. Policy **17**, S93–S110 (2017)
17. Tulkens, H.: Economics, Game Theory and International Environmental Agreements: The Ca' Foscari Lectures. World Scientific Publishing (2019)