An Opinion dynamics coupled to an environmental dynamics

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Context

This study examines how social influence and environmental feedback shape group behavior in a multi-agent system. Each agent's opinion affects their emissions, which impact the environment, creating a feedback loop. As the environment changes, agents adjust their opinions based on both social interactions and environmental signals.

The model combines social and environmental dynamics to explore how these factors together influence collective behavior.

Multi-agents model

We consider a system of N agents interacting through a strongly connected directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where each agent $i \in \mathcal{V}$ holds at time k an opinion $\theta_i(k) \in [-1, 1]$. The opinion reflects the agent's behavior towards the environment, with $\theta_i(k) \approx -1$ indicating pro-environmental behavior, and $\theta_i(k) \approx 1$ representing environmental indifference.

Environment dynamics

- **Emissions**: Agents' emissions depend on their opinions via the function $E(\theta_i)$, with $\theta_i \in [-1, 1]$ determining emission levels $[e_{\text{min}}, e_{\text{max}}]$.
- **Environment Dynamics**: The environment's state $p(k)$ evolves based on cumulative emissions:

$$p(k+1) = \gamma p(k) + \sum_{i=1}^{N} E(\theta_i(k))$$

where $0 < \gamma < 1$ is the natural pollution decay rate.

Opinion Dynamics

Each agent adjusts their opinion according to:

$$\theta_i(k+1) = \theta_i(k) + \left(1 - \theta_i(k)^2\right) \left[\beta s_p(\bar{p} - p(k)) + \frac{1-\beta}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} s_\theta(\theta_j(k)) - \theta_i(k) \right]$$

The term $(1 - \theta_i(k)^2)$ represents **stubbornness**, making agents with extreme opinions, i.e. $\theta_i(k) \approx \pm 1$, less likely to change.

- **Environmental Influence**: Agents respond to the environmental state $p(k)$, which they perceive relative to a threshold $\bar{p}$ via $s_p : \mathbb{R} \to [-1, 1]$, an increasing function.
- **Social Influence**: Agents also adapt their opinions based on the actions of their neighbors $\mathcal{N}_i$ via $s_\theta : [-1, 1] \to [-1, 1]$, an increasing function.

The parameter $0 < \beta < 1$ is the trade-off between the two influences.

Coupled dynamics

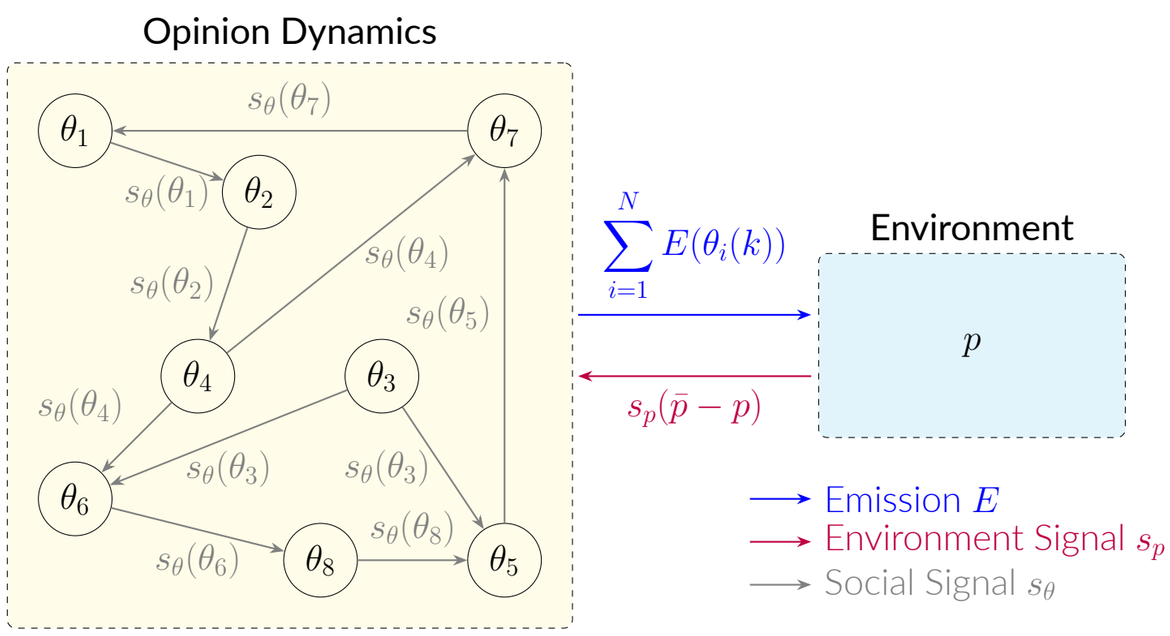


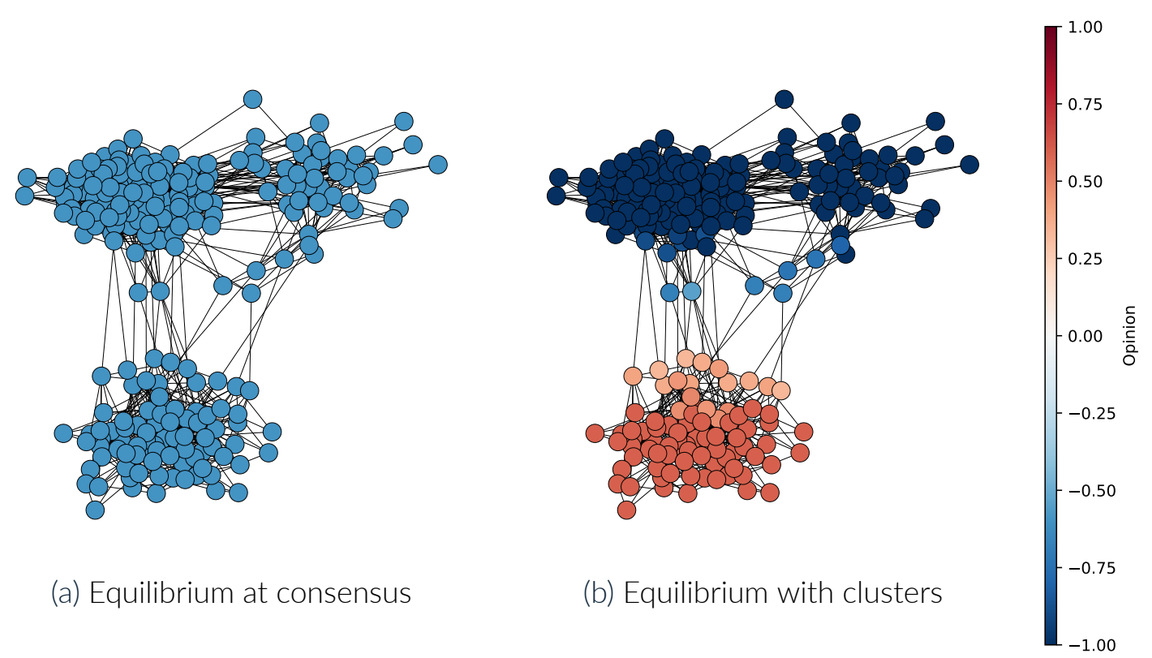
Figure 1. Diagram of the coupled dynamics.

Asymptotic behaviors

There is 3 different regimes, depending of β and γ:

Equilibrium:

- Stable equilibria can be either at consensus or with clusters:

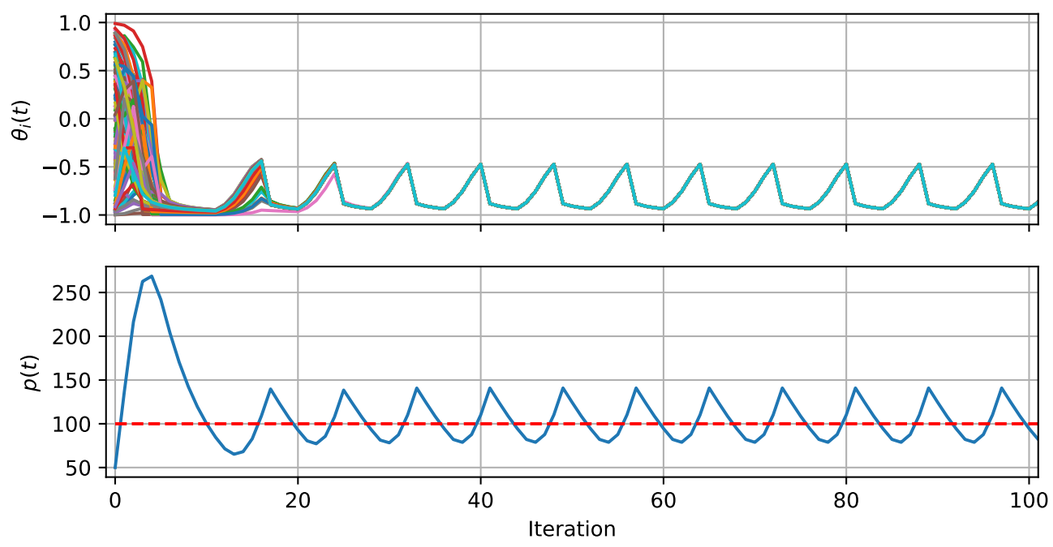


(a) Equilibrium at consensus (b) Equilibrium with clusters

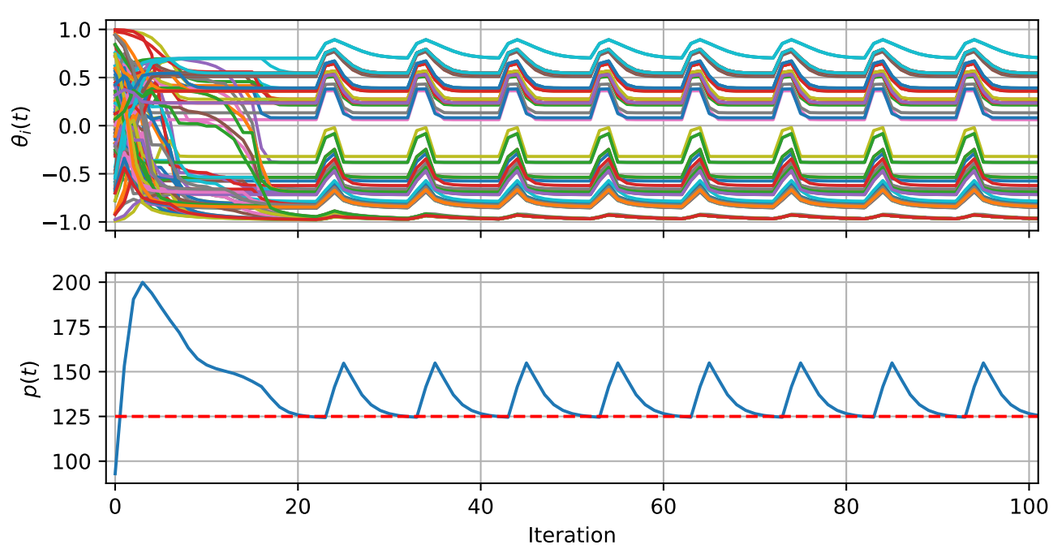
- Equilibria with extreme opinions are unstable, i.e. if $\theta_i = \pm 1 \Rightarrow$ unstable.

Limit cycles:

- Opinion and environment oscillate like a prey-predator system.
- The synchronized opinion manifold is forward invariant:



- Cluster structures can also oscillate and remain robust to oscillation of p.



Chaotic:

- Chaotic behaviors appear in which synchronization cannot be guaranteed.
- Trajectories converge to a strange attractor.

References

[1] A. Couthures, T. Mongaillard, V.S. Varma, S. Lasaulce, and I.C. Morărescu. Analysis of a continuous opinion and discrete action model coupled with an external dynamics. In *2024 European Control Conference (ECC)*, pages 3392–3397, 2024.