On the impact of cross-country imitation on climate change: A game-theoretical analysis

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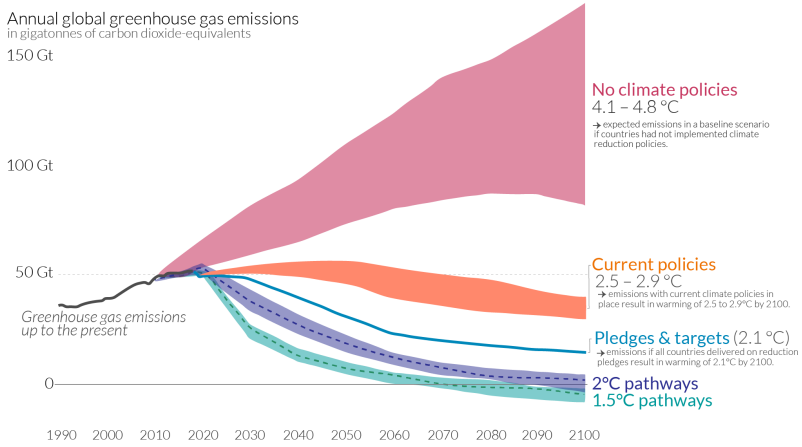


Introduction

Global greenhouse gas emissions and warming scenarios

Our World in Data

– Each pathway comes with uncertainty, marked by the shading from low to high emissions under each scenario.
– Warming refers to the expected global temperature rise by 2100, relative to pre-industrial temperatures.



Data source: Climate Action Tracker (based on national policies and pledges as of November 2021).
OurWorldinData.org – Research and data to make progress against the world's largest problems.

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Rethinking Decarbonization Strategies

Naive question at hand:

- If governments want to reduce temperature increase why don't they change emission policy?

Our Focus:

- Game-theoretic approach to assess the lack of coordination strategies and qualitative behaviors.

$$\underline{\text{Game Theory}} + \underline{\text{Climate Modeling}} = \textbf{Answer? Solution?}$$

State of the Art in Decarbonization Efforts

Climate Science

Models and projections

[IPCC, 2021]
[Geoffroy et al., 2013]
[Smith et al., 2018]
[Leach et al., 2021]
[Joos et al., 2013]
...

Economics

General equilibrium

Carbon pricing

[Nordhaus and Yang, 1996]
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...

Game Theory

Prisoner Dilemma without simulation

Coalitions

[Finus, 2002]
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Some do links:

[Dietz et al., 2021], [Tulkens and Eyckmans, 1999], [Tulkens, 2019], [Faulwasser et al., 2018],...

Observation: While multiple disciplines address decarbonization, there is still a crucial lack of inter-disciplinary coordination.

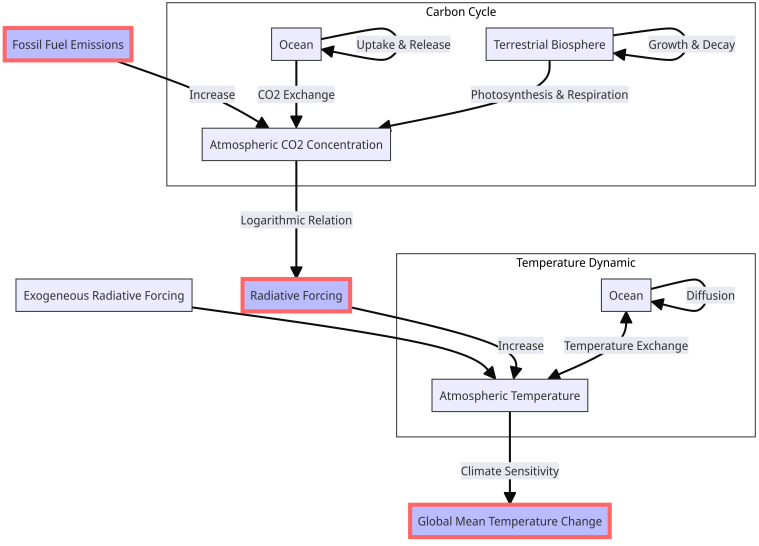
Outline

1. Models
   Climate Model (CM)
   Integrated Assessment Models (IAM)

2. General Game

3. Imitation Game

Climate Model (CM)



Simplifying the Climate Model

We use linear systems, but the approach can be generalized.

Carbon Cycle: state C

$$\begin{cases} C_{t+1} = \mathbf{A}_C C_t + b_C E_t \\ C_t^{\mathrm{AT}} = d_C^\top C_t \end{cases}$$

Temperature Dynamic: state θ

$$\begin{cases} \theta_{t+1} = \mathbf{A}_\theta \theta_t + b_\theta F_{t+1} \\ \theta_t^{\mathrm{AT}} = d_\theta^\top \theta_t. \end{cases}$$

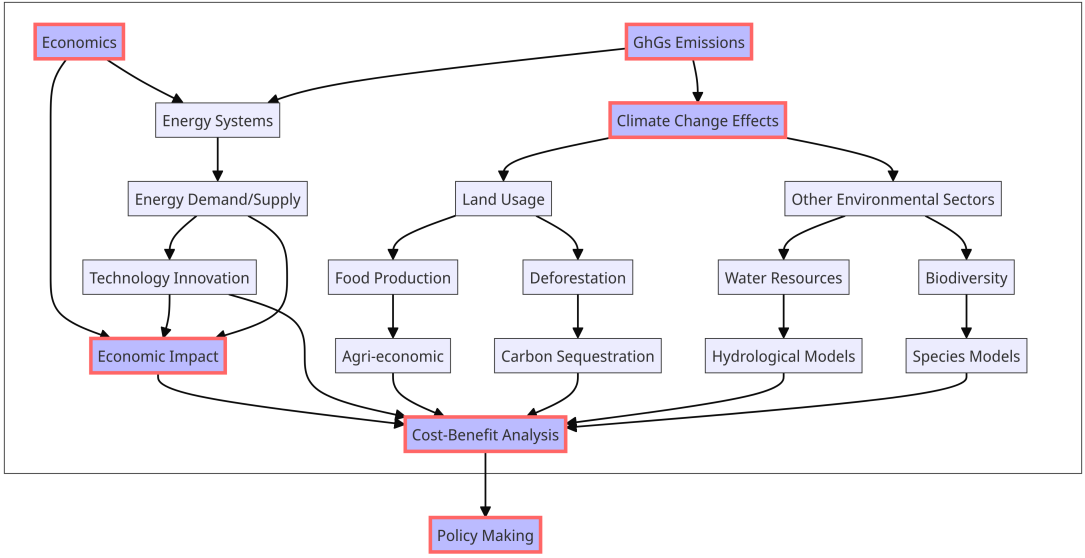
N.B: "AT"stands for atmospheric. From 2020 to 2100, with $\Delta t = 5$: $T = 16$ steps.

Logarithm relation:

$$F_t = F_{2\times \mathrm{CO}_2} \log_2 \left(\frac{C_t^{\mathrm{AT}}}{C_{\mathrm{ref}}^{\mathrm{AT}}} \right) + F_{t,nonCO_2}.$$

Caution: The chosen model influence the behavior of the players.

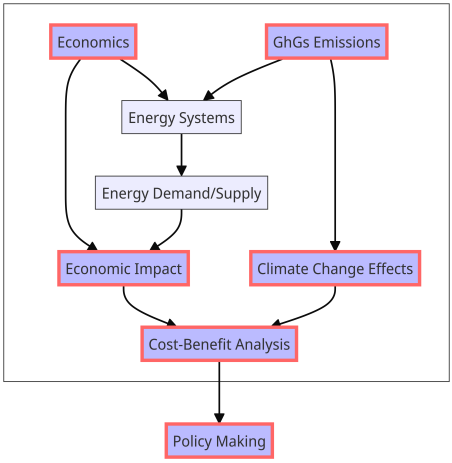
Integrated Assessment Models (IAMs)



Simplifying the IAM

IAMs:

- Couple climate science with economics.
- Estimate the impact of climate policies.
- Evaluate costs and benefits of action/inaction.
- Really complex and often not well calibrated



Key Takeaway: Links Benefits from energy consumption and Damages from the climate.

Outline

1. Models

2. General Game
   - A Model from Literature
   - Our model adaptations
   - Methodology
   - Efficiency of Nash equilibrium

3. Imitation Game

From Literature: The Basic Game Model

Simplifying to a Basic Game

Since IAMs are complex and often opaque, we return to a fundamental **one shot** game model from the literature[a].

[a] [Finus, 2002],[Tulkens, 2019],[Chander, 2018]

- Game with N countries, country n's action a_n are CO_2 emissions.
- Payoff for n is a function of its own action and the sum of all:

$$u_n(a_n, a_{-n}) = B_n(a_n) - D_n\left(\sum_{k=1}^{N} a_k\right),$$

- $B_n(a_n)$: Economic benefits country n from its emissions.
- $D_n\left(\sum_{k=1}^{N} a_k\right)$: Damages to country n from the total emissions.

Our model adaptations

Addapt:

- a_n action of player $n \in \mathcal{N} := \{1, \ldots, N\}$,
  $\Rightarrow a_n \in [0, e_n^{max}] = \mathcal{A}_n$, $\mathcal{A} = \prod_{n \in \mathcal{N}} \mathcal{A}_n$

- $\theta^{\mathrm{AT}} : \mathcal{X} \times \mathcal{A}$: Atmospheric temperature from climate model.
- $x = (C, \theta) \in \mathcal{X}$ the state, describing the climate

- Payoff function
$$u_n(x, a) = B_n(a_n) - w_n D\left(\theta^{\mathrm{AT}}(x, a)\right)$$
- D **is now global!** (i.e. not D_n anymore)
- $w_n \geq 0$ multiplier of damage to n, $D_n \simeq w_n D$[1]

[1] [Tulkens, 2019]

About the Model

New Assumptions:

- On B_n and D:

$$B_n(a_n) : \text{Increasing, quasi-Concave} \quad (B_n' > 0, B_n'' \leq 0)$$
$$D\left(\theta_t^{\mathrm{AT}}\right) : \text{Increasing, Convex} \quad (D_n' > 0, D_n'' \geq 0)$$

Interests:

- With a climate model
- Simulation of behaviors possible
- **What literature do without noticing?**[2]

[2] [Tulkens, 2019]

Complete Formalism of Our Game

Let us include time dependence:

Definition (Planning game)

For a given time $T \in \mathbb{N}$, for any $x_1 \in \mathcal{X}$, $(a_n^T, a_{-n}^T) \in \mathcal{A}^T$, player n's utility $U_n^T : \mathcal{X} \times \mathcal{A}^T \to \mathbb{R}$ is

$$U_n^T(x_1, a_n^T, a_{-n}^T) = \sum_{t=1}^{T} u_n(x_t, a_{nt}, a_{-nt}).$$

The instantaneous utility is given by a benefit function $B_n : \mathcal{A}_n \to \mathbb{R}$ minus a weighted damage $D : \mathbb{R} \to \mathbb{R}$, for some weights $w_n > 0$, $\forall n \in \mathcal{N}$ as follows

$$u_n(x_t, a_{nt}, a_{-nt}) = B_n(a_{nt}) - w_n D\left(\theta_{t+1}^{\mathrm{AT}}\right).$$

Now: Existence? Uniqueness? Conditions? of Nash Equilibrium

Potential Game?

Introducing some particular games:

Definition (Weighted Potential Game)

Game $(\mathcal{N}, \mathcal{A}, (u_n)_{n\in\mathcal{N}})$ is a weighted potential game i.f.f. there exists a function $\phi : \mathcal{A} \longrightarrow \mathbb{R}$ and (ω_n) positive weights, s.t, for all $n \in \mathcal{N}$, $a_n, \tilde{a}_n \in \mathcal{A}_n$ and $a_{-n} \in \mathcal{A}_{-n}$,

$$u_n(a) - u_n(\tilde{a}_n, a_{-n}) = \omega_n \left[\phi(a) - \phi(\tilde{a}_n, a_{-n})\right].$$

Is our game potential?

Our one-stage game is! $\phi(x, a) = \sum_{n=1}^{N} \frac{B_n(a_n)}{w_n} - D\left(\theta^{\mathrm{AT}}(x, a)\right)$.
But what about the planning game?

State transitive Potential game

Definition (State transitive Potential Game)

A game is state transitive weighted potential if there exists $\phi : \mathcal{X} \times \mathcal{A} \to \mathbb{R}$ and positive weights $(\omega_n)_{n \in \mathcal{N}}$ such that the following holds for any $a \in \mathcal{A}$, $\forall x, \tilde{x} \in \mathcal{X}$,

$$u_n(x, a) - u_n(\tilde{x}, a) = \omega_n \left(\phi(x, a) - \phi(\tilde{x}, a)\right), \quad \forall n \in \mathcal{N}.$$

Our one-stage game is state transitive!
Then our Planning game is too with following potential:

$$\Phi^T(x_1, a^T) = \sum_{t=1}^{T} \phi(x_t, a_t).$$

$\Rightarrow$ **Free existence of Pure NE in the planning game.**

Conditions for uniqueness

Proposition (Uniqueness Conditions)

Assuming as before. $\mathbf{C}^{\mathrm{AT}}(x)$, $\Theta^{\mathrm{AT}}(x)$ *the possible state from* x.

- $\forall C^{\mathrm{AT}}, \tilde{C}^{\mathrm{AT}} \in \mathbf{C}^{\mathrm{AT}}(x)$, *such that* $C^{\mathrm{AT}} < \tilde{C}^{\mathrm{AT}}$, *we have*

$$\frac{D'(\tilde{\theta}^{\mathrm{AT}})}{D'(\theta^{\mathrm{AT}})} \geq \frac{\tilde{C}^{\mathrm{AT}}}{C^{\mathrm{AT}}}, \qquad \textbf{\textit{(Damage must be fast exp-like)}}.$$

- $\exists n \in \mathcal{N}$ *such that* $\forall a_n, \tilde{a}_n \in \mathcal{A}_n$, $a_n < \tilde{a}_n$, *we have*

$$\frac{B_n'(\tilde{a}_n)}{B_n'(a_n)} \leq \frac{C^{\mathrm{AT}}}{\tilde{C}^{\mathrm{AT}}}, \qquad \textbf{\textit{(Benefits must be slow log-like)}}.$$

- D *is twice differentiable and for* $x \in \mathcal{X}$, $\forall \theta^{\mathrm{AT}} \in \Theta^{\mathrm{AT}}(x)$

$$D''(\theta^{\mathrm{AT}}) \geq D'(\theta^{\mathrm{AT}})/\tilde{b}_\theta, \qquad \textbf{\textit{(Damage must be fast exp-like)}}.$$

Why such conditions are important?

These conditions are not present in the literature.
They incentivise to reduce emissions.

Assumption (Benefits on same scale than damage)

For all $n \in \mathcal{N}$, $t \in \mathcal{T}$, $B'_{nt}(e_{nt}^{\max}) = 0$ or be negligible compared to the damages induced by players' emissions.

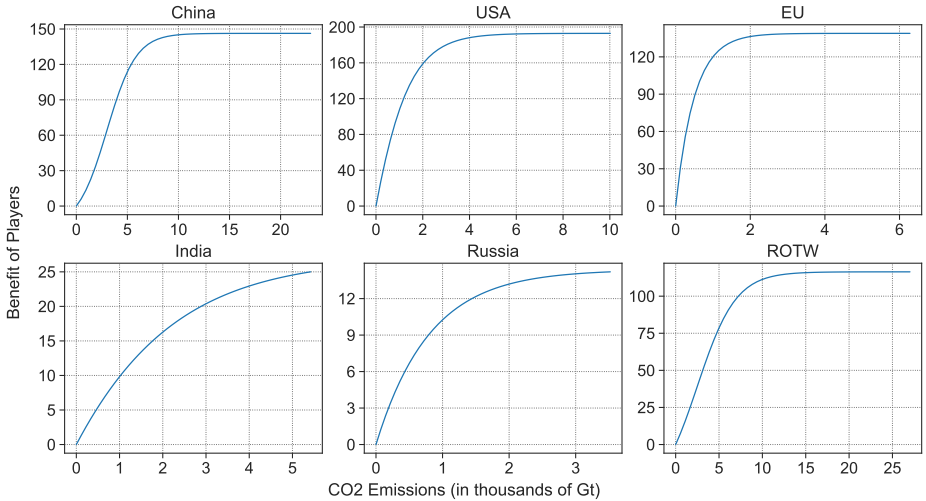
Literature, mainly quadratic damages:

- $D(X) = 1.145X^2$,
- $D(X) = 0.267X^2$,
- $D(X) = 1.1X^2 - 2.46X$,

Quadratic is too weak!
It does not satisfy the condition on the damage at all!

Condition for the simulation: Fixed parameters

Benefit function: in (10^{11} US$ 2011)



How the game is played

We need to compare efficiency in 2100:
$t_0 = 2020$, $t_{\max} = 2100$, $\Delta t = 5 \Rightarrow T = 16$

Planning game: 1 game with $T = 16$

Repeated One-stage: 16 games with $T = 1$:
$\Rightarrow$ at t the game is played for **1** step with state $x_t = (C_t, \theta_t)$
$\Rightarrow$ the actions provide a $x_{t+1} = (C_{t+1}, \theta_{t+1})$

Receding horizon: 16 games with $T = 16$
$\Rightarrow$ at t the game is played for **16** step with state $x_t = (C_t, \theta_t)$
$\Rightarrow$ the actions provide a $x_{t+1} = (C_{t+1}, \theta_{t+1})$

Outcomes change a lot depending on the ways you play the game

One-Stage game

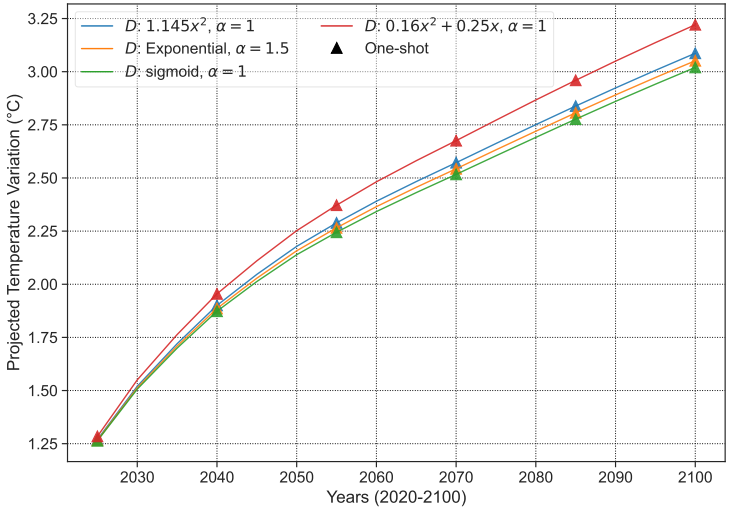


Figure: Temperature Trajectory from 2020 to 2100 (C°)

One-Stage and Planning games

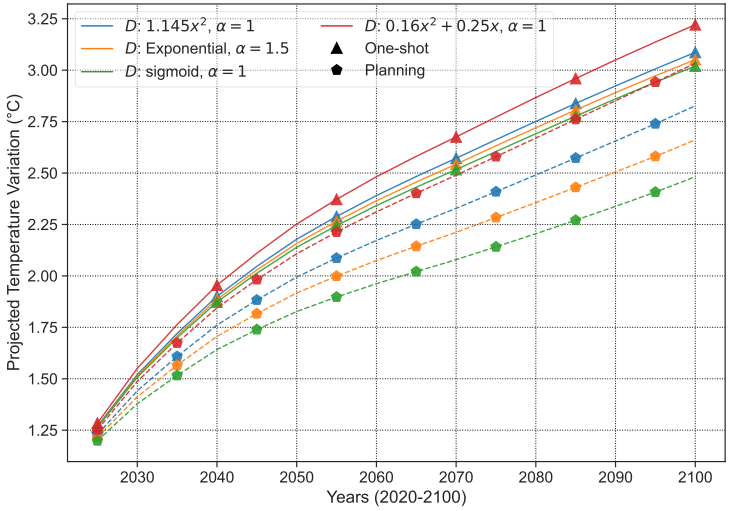


Figure: Temperature Trajectory from 2020 to 2100 (C°)

All games Temperature

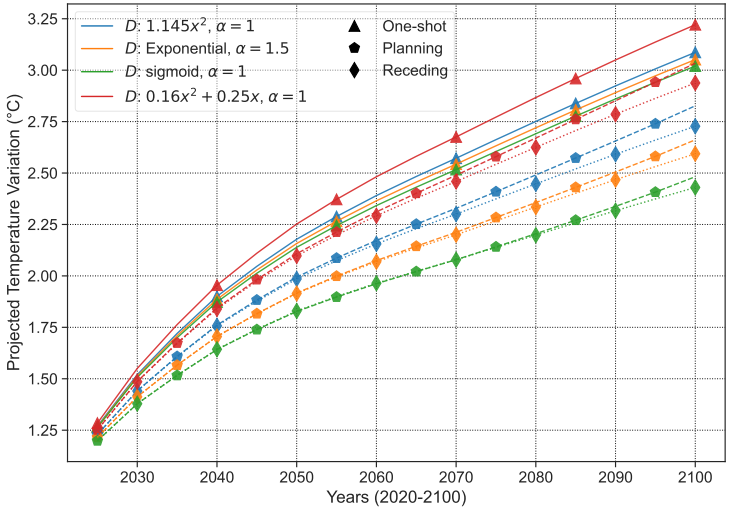


Figure: Temperature Trajectory from 2020 to 2100 (C°)

All games Temperature - Two damage function

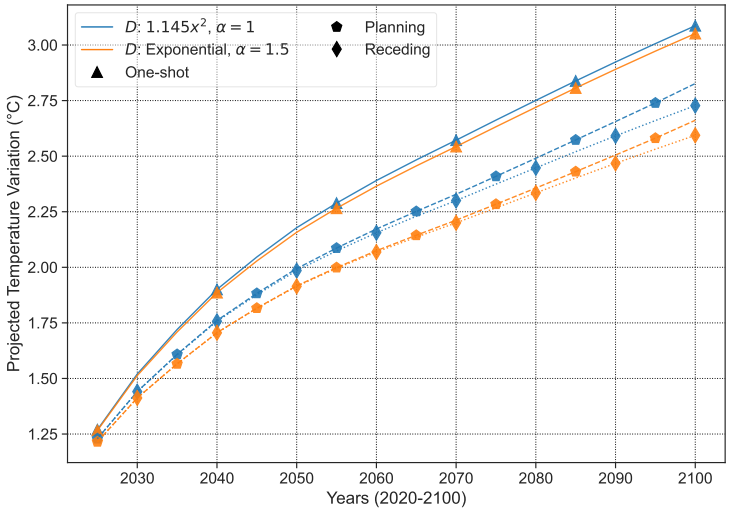


Figure: Temperature Trajectory from 2020 to 2100 (C°)

All game Actions - Two damage function

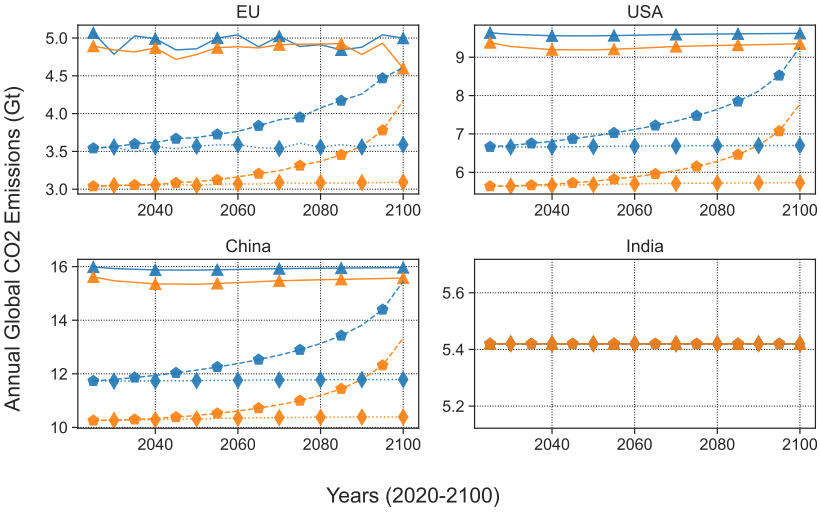


Figure: Actions of EU, USA, China and India (GtCO2).

What can we say?

We observe[3]:

- The strongest the damage function the lower the temperature.
- We always have θ^{AT}: One-stage $>$ Planning $>$ Receding.
- We always have Emissions: One-stage $>$ Planning $>$ Receding.
- Action behaviors per game
  - In One-stage action almost does not change
  - In Planning, it increases at the end (do not consider period after 2100)
  - In Receding, it stagnates but much lower
- There is players (like India) that never change their actions: e_{max} is dominating strategy.

Intuitive results are verified in our model.

[3] Under constant action set / benefit function / $\mathrm{GDP}^{\mathsf{max}}$

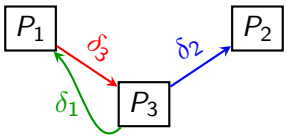
Outline

1. Models
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3. Imitation Game
   New utility in the One-Shot game
   Simulation

Imitation in the One-shot game

Changing the game:

Now agents influence each other,



- Taking an imitation term associated with a influence graph,

$$\mathrm{Imi}_n(a) = \delta_n e_n^{\max} \sum_{m \in \mathcal{V}_n} e_m^{\max} \left(\frac{a_n}{e_n^{\max}} - \frac{a_m}{e_m^{\max}} \right)^2$$

- Adding to u_n the imitation term:

$$u_n(x, a) := B_n(a_n) - w_n D\left(\theta^{\mathrm{AT}}(x, a)\right) - \delta_n \mathrm{Imi}_n(a).$$

What can we say in this case?

- It's not anymore a potential game, but we can exhibit condition for existence of Pure NE: D_n exponentially increasing with

$$B_n''(a_n) < 2\left(\sum_{m\in\mathcal{V}_n}\frac{e_m^{\max}}{e_n^{\max}} - \frac{N+2}{2}\right)\delta_n - \sum_{n=1}^{N}\delta_n, \quad \forall n \in \mathcal{N}$$

- In the Quadratic case (B_n and D quadratic), with a complete interaction graph, we can have an explicit formulation of the pure NE.

Global influence vs clustered influence

Two influence network structure: fully connected vs Sparse Directed graph

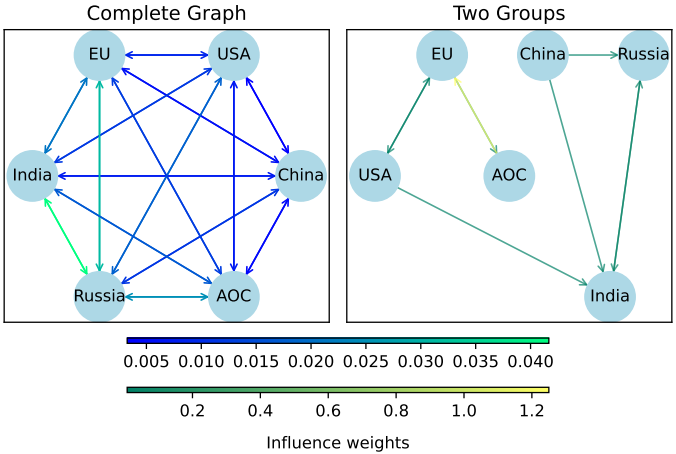


Figure: The two topology taken. Edges are weighted.

Global influence vs clustered influence

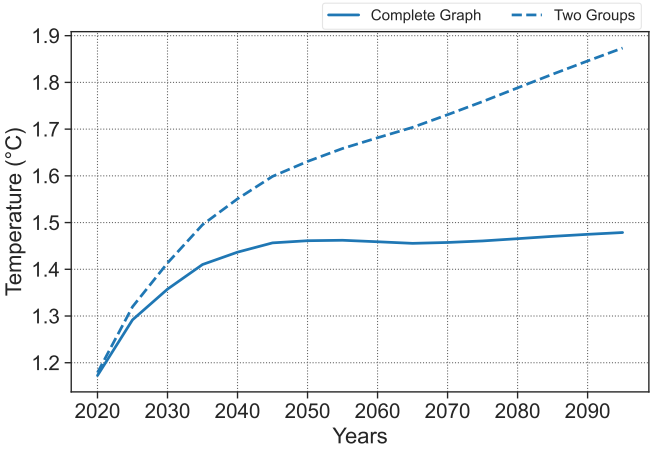


Figure: Temperature evolution CO_2 at NE for the two topology.

Global influence vs clustered influence

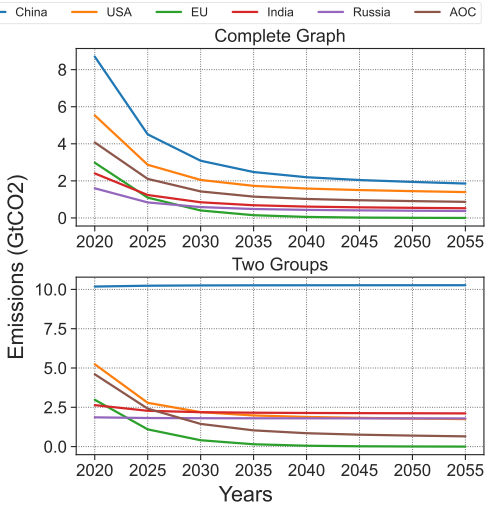


Figure: CO2 emission at NE for both topology.

Conclusion

- **Contributions**
  - New structured game including climate model and imitation
  - Equilibrium analysis: existence and explicit characterization in quadratic case
  - Important conditions for uniqueness of NE that damage functions must to satisfy
  - Approach that work with any climate model
  - Simulator: allowing to find equilibrium to see behaviors of players and impacts on climate, available on Github (Beta) [Couthures, 2023]

- **Future research topics**
  - Opinion dynamic: to connect player with economic relation
  - Include mediator: United Nation with goal to reach target temperature
  - Better economy modeling: dynamical system with dynamic action sets
  - Add imitation in the planning settings

To take away

Key ideas:

- Our game is potential for any Climate Model
- Existence of Nash Equilibria for very weak conditions
- Uniqueness of Nash Equilibrium is related on the structure of Benefit versus Damage
- Quadratic damage function are too weak to change the behaviors of players in simulation

Comparison to utilitarian Social Optimum (SO)

What is utilitarian Social Optimum?

The social optimum is the configuration of actions that maximize the sum of all players' utilities. It is a common object to look at in Game Theory - as a reference to assess how much selfish behaviors may induce a loss of optimality.

Definition (Social Optimum problem)

The social optimum problem, starting from a initial climate state $x \in \mathcal{X}$, can be mathematically represented as:

$$\max_{a^T \in \mathcal{A}^T} \bar{U}^T(x, a^T) := \sum_{n=1}^{N} U_n^T(x, a_n^T, a_{-n}^T)$$

In the following, only look at One-shot and Receding

NE versus SO: temperature

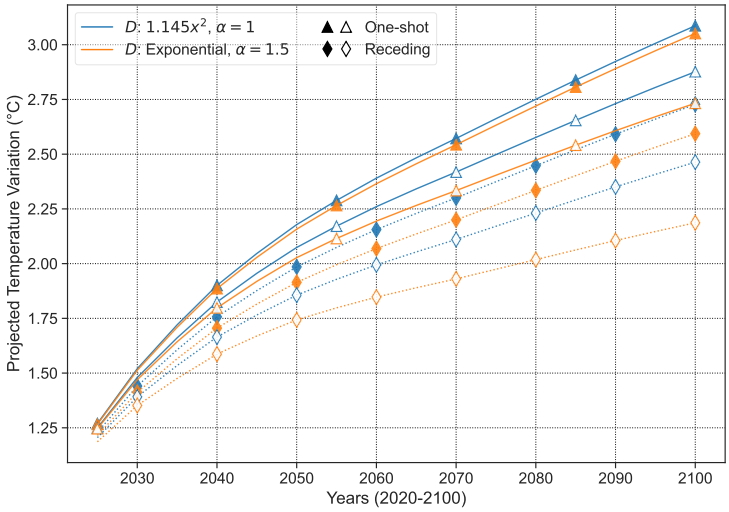


Figure: θ^{AT}: Nash Equilibrium $>$ Social optimum

NE versus SO: actions

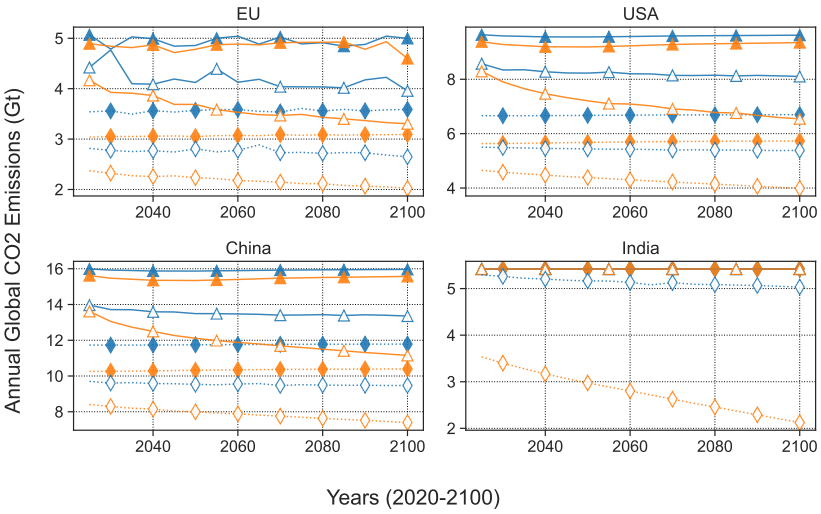


Figure: Action - Nash Equilibrium > Social optimum

NE versus SO: utilities

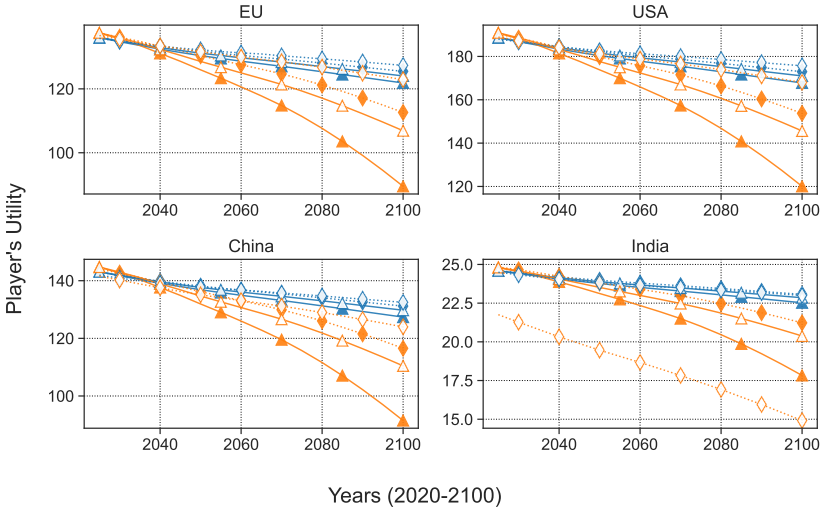


Figure: Utility - better with cooperation for almost all player.

What can we say?

We observe[4]:

- We always have θ^{AT}: Nash Equilibrium $>$ Social optimum.
- We always have Emissions: Nash Equilibrium $>$ Social optimum.
- Utility are better with cooperation for almost all player
  $\Rightarrow$ for One-shot and Receding game
- At the end of century there is a big difference of utility between NE and SO
- **Receding SO isn't interesting solution compare to NE for India** $\Rightarrow$ it cannot be a Nash equilibrium
- There is players (like India) that never change their actions: e_{max} is dominating strategy except in social optimum.

Cooperation: not always profitable on short term for CO2 based economies.

[4] Under constant action set / benefit function / $\mathrm{GDP}^{\mathsf{max}}$

Efficiency Solution concept for two players

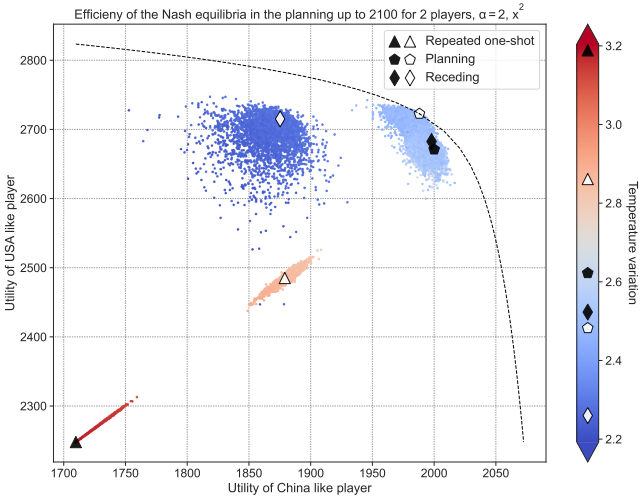


Figure: Aggregated USA and China. Efficiency of solution concepts individually.

Efficiency Solution concept for two players

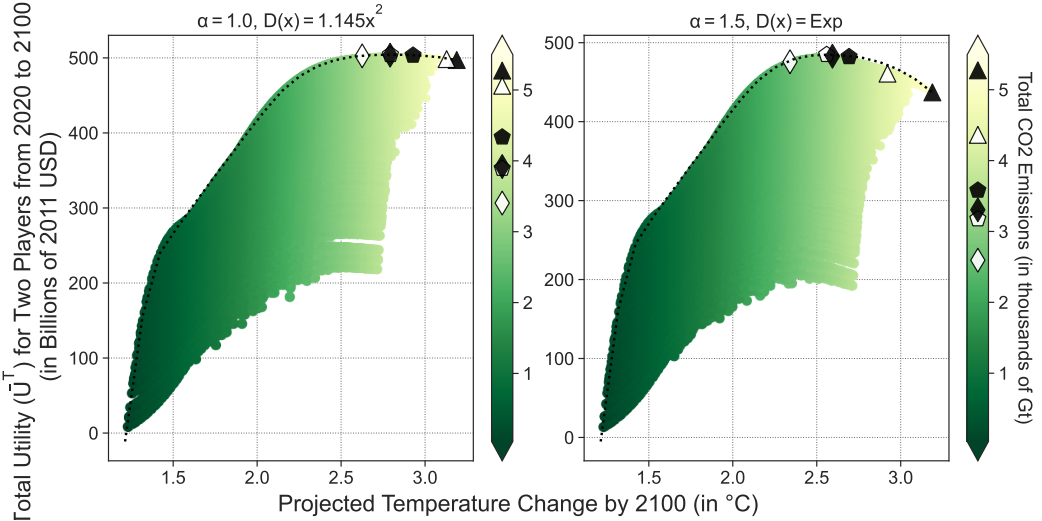


Figure: Comparison of strategies efficiency from 2020 to 2100 for different damage function in aggregated way

State of the Art in Decarbonization Efforts

Climate Science

Models and projections

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General equilibrium

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Game Theory

Prisoner Dilemma without simulation

Coalitions

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Some do links:

[Dietz et al., 2021], [Tulkens and Eyckmans, 1999], [Tulkens, 2019], [Faulwasser et al., 2018],...

Observation: While multiple disciplines address decarbonization, there is still a crucial lack of inter-disciplinary coordination.

About the Literature Model

Assumptions:

- $B_n(0) = 0$ and $D_n(0) = 0$: No benefit or damage without emissions.
- Benefits[5] and damages[6]:

$$B_n(a_n) : \text{Increasing, Concave} \quad (B'_n > 0, B''_n \leq 0)$$
$$D_n\left(\sum a_k\right) : \text{Increasing, Convex} \quad (D'_n > 0, D''_n \geq 0)$$

Issues:

- Without a climate model
- No simulation possible that capture climate change
- No equilibrium uniqueness guarantee nor determination

[5] [Finus, 2002]
[6] [Weitzman, 2010]

Fixed parameters

- $N = 6$
- Characteristics of players

Player	e_n^{max} (GtCO2)	$\mathrm{GDP}_n^{\max}$ (10^{11} US$ 2011)	w_n
China	11	146.3	1.1847
USA	7	192.9	1.1941
EU	4	138.9	1.1248
India	3	25	0.9074
Russia	2	14.2	1.2866
ROTW	5	116.4	1.1847

- Climate model:
  Carbon Cycle [Joos et al., 2013] + Temperature Dynamic [Geoffroy et al., 2013] (best fit IPCC prediction)

Discretized “Variable”

Candidates damage function : in (% $\mathrm{GDP}^{\max}$)

