ICTEAM Newcomer Seminar: Presentation, Previous and Current Research

Anthony Couthures

Promotors: Gianluca Bianchin and Julien Hendrickx

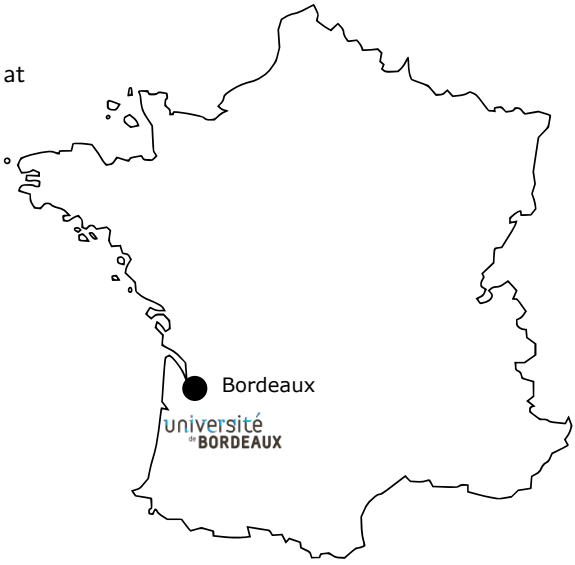
6 October 2026

UCLouvain

SIDDARTA

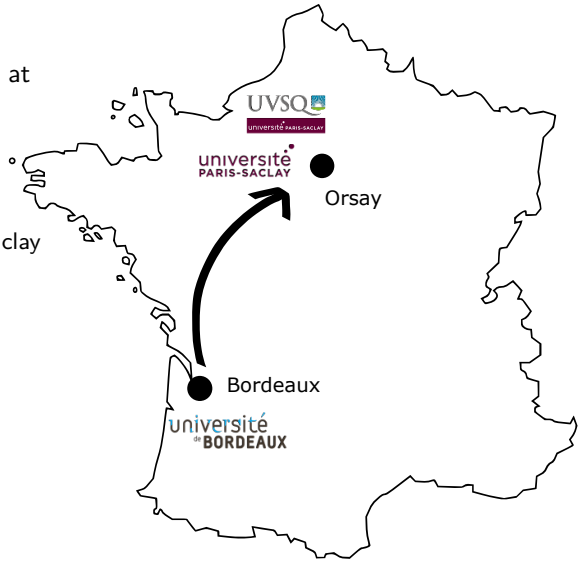
Academic journey

- **Bachelor:** in Fundamental Mathematics, at Bordeaux



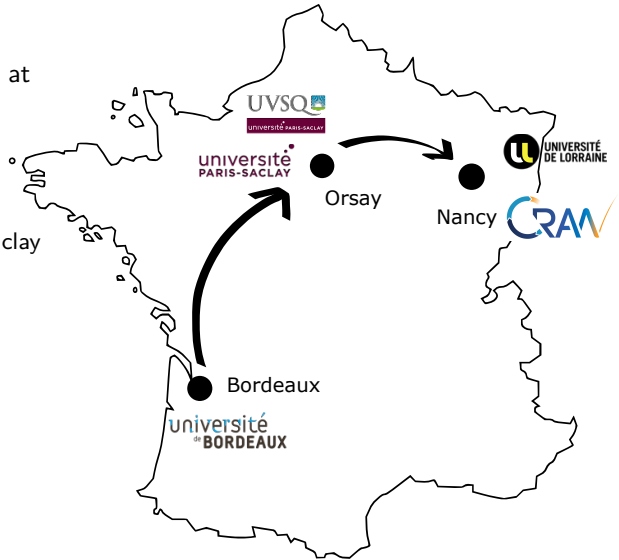
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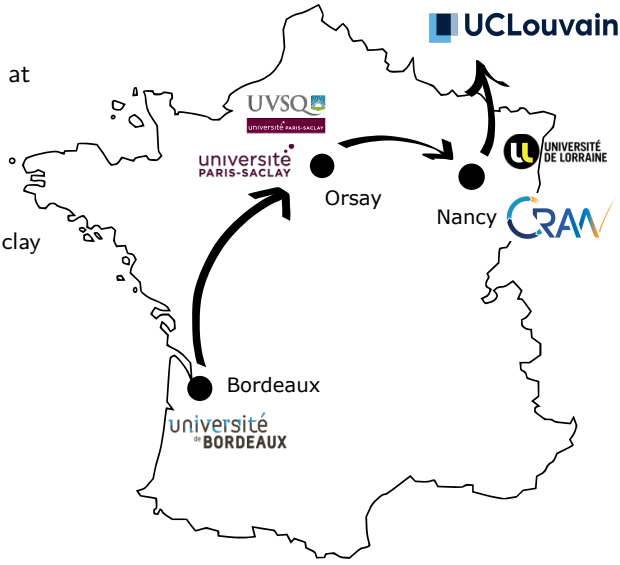
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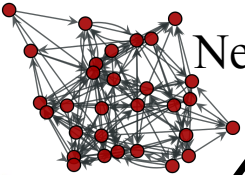


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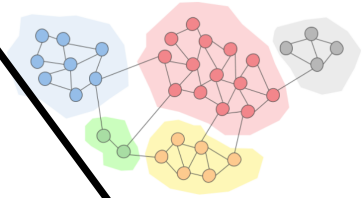


Research interests



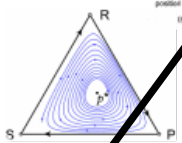
Networks

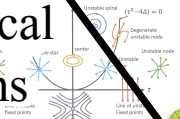


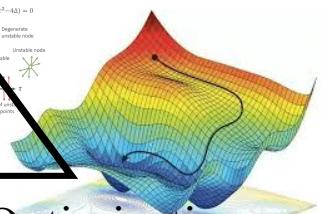




Dynamical Systems

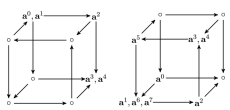








Game Theory



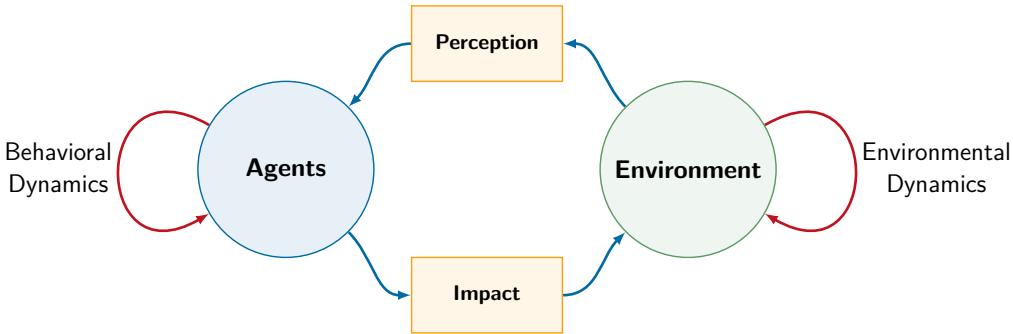
Optimization

Outline

1. PhD: Opinion Dynamics coupled with an Environment
2. Postdoc: Performance Estimation Problem for Control

How to model this socio-environmental system?

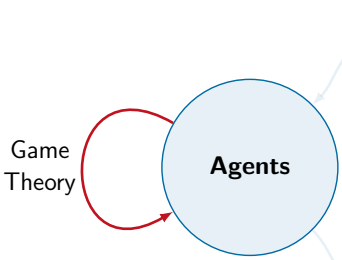
A mathematical sociological modeling problem:



Dynamics Agents have an **impact** on a **dynamic environment**, their behavior is influenced by their **perception** of the environment.

How to model this socio-environmental system?

A mathematical sociological modeling problem:



Two Modeling Approaches

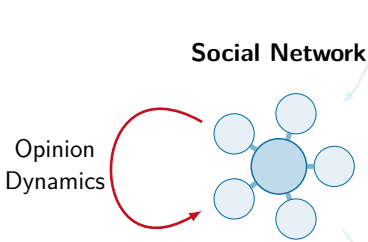
1. **Strategic**: Optimizers of their own utility.

Publications with Strategic Agents (Game theory):

- "A Quadratic Static Game Model for Assessing the Impact of Climate Change", Mroué, Couthures, et al. (2024)
- "On the Impact of Cross-Country Imitation on Climate Change", Mroué, Couthures, et al. (2025)
- "International Climate Policy as a Potential Game", Couthures et al. (sub2026)

How to model this socio-environmental system?

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Two Modeling Approaches

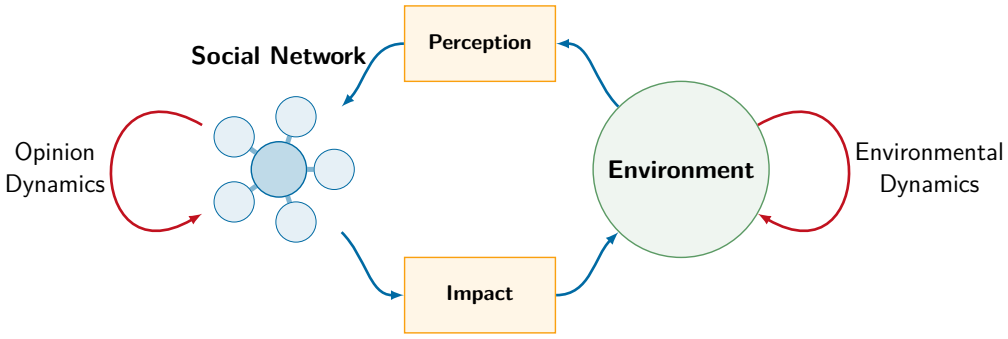
1. **Strategic**: Optimizers of their own utility.
2. **Non-strategic**: Evolving with social influence.

Publications with non-strategic agents (Opinion Dynamics with environment):

- "Analysis of a CODA Model Coupled with an External Dynamics", Couthures et al. (2024a)
- "Analysis of an OD Model Coupled with an External Environmental Dynamics", Couthures et al. (2024b)
- "Bifurcation Analysis of an OD Model Coupled with an Environmental Dynamics", Couthures et al. (2025)

How to model this socio-environmental system?

A mathematical sociological modeling problem:



For this talk, we will focus on **understanding the behavior of a group of agents interacting with an environment** in the context of **opinion dynamics on graphs**.

Our Model

- **Opinion Dynamics:** for agent i, the opinion dynamics is given by:

$$\dot{x}_i = -x_i + \underbrace{\beta r(E)}_{\text{Env. Feedback}} + \underbrace{(1-\beta)\frac{1}{d_i}\sum_{j\in\mathcal{V}} a_{ij}s(x_j)}_{\text{Network Social Influence}}$$

- $\beta \in [0,1]$: Attention parameter.
- $x_i \in [-1,1]$: Agent i opinion.

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Two main improvements over the literature

Only 1 agent vs the env.:

- (Weitz et al. 2016)
- (Tilman et al. 2020)
- (Ito and Yamamichi 2024)

Polarization on static graphs need signed interaction:

- (Altafini 2013)
- (Gray et al. 2018)
- (Bizyaeva et al. 2023)

→ Naomi Leonard's group

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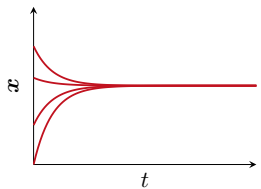
- **Environmental dynamics:** Physical process driving the evolution of the environment.

$$\tau_e \dot{E} = -\gamma E + u(\boldsymbol{x})$$

  - τ_e: time scale.
  - $u(\boldsymbol{x})$: Agents' collective impact.
  - γ: Restoring force of the environment.

Modeling the evolution of opinions

The behaviors our model can capture are:



(a) Constant consensus

(b) Constant polarized

(c) Oscillating consensus

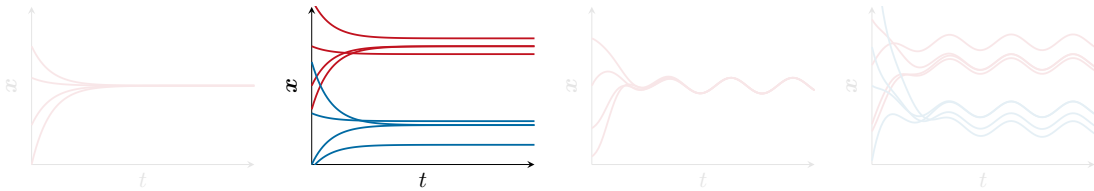
(d) Oscillating polarized

Captured behaviors: Constant consensus

Example: Scientific consensus on the existence of global warming.

Modeling the evolution of opinions

The behaviors our model can capture are:



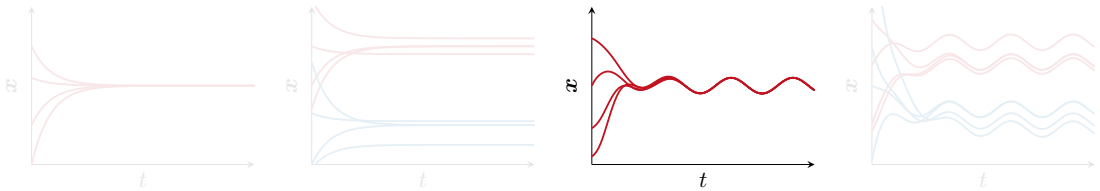
(a) Constant consensus (b) Constant polarized (c) Oscillating consensus (d) Oscillating polarized

Captured behaviors: Polarized behavior

Example: Entrenched public division regarding nuclear energy or carbon taxation.

Modeling the evolution of opinions

The behaviors our model can capture are:



(a) Constant consensus (b) Constant polarized (c) Oscillating consensus (d) Oscillating polarized

Captured behaviors: Oscillating consensus

Example: Collective risk perception fluctuating directly with environmental crises.

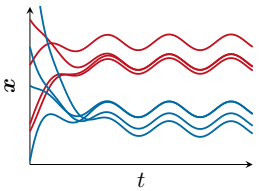
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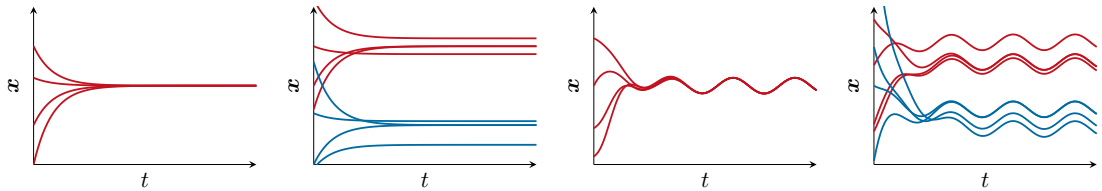
(d) Oscillating polarized

Captured behaviors: Oscillating polarized

Example: Partisan divides where the intensity fluctuates with political cycles or events.

Modeling the evolution of opinions

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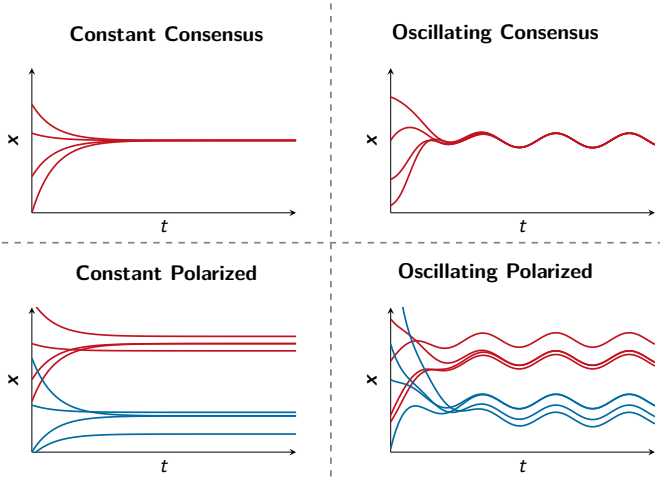


(a) Constant consensus (b) Constant polarized (c) Oscillating consensus (d) Oscillating polarized

Questions: How do the different behaviors emerge?

1. How do we shift **from consensus to polarization**?
2. How do we shift **from constant to oscillating consensus**?

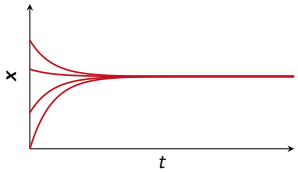
Thesis - Results



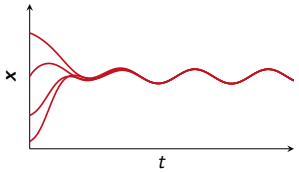
Thesis - Results

Increase β (Prey-Predator System)

Constant Consensus



Oscillating Consensus



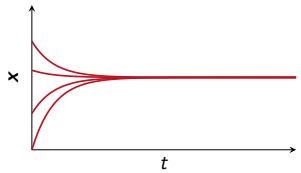
Constant Polarized

Oscillating Polarized

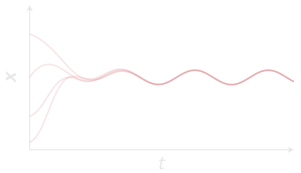
Thesis - Results

Amplification of opinion
×
Reduce Network Connectivity

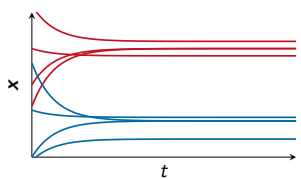
Constant Consensus



Oscillating Consensus

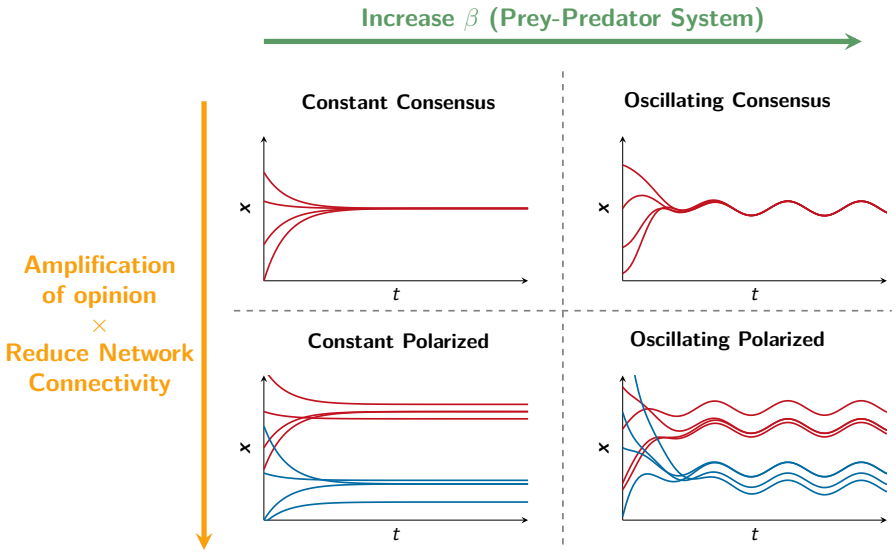


Constant Polarized



Oscillating Polarized

Thesis - Results



Outline

1. PhD: Opinion Dynamics coupled with an Environment
2. Postdoc: Performance Estimation Problem for Control

The Performance Estimation Problem (PEP)

What is PEP?

Framework to find the **worst-case performance of algorithms** as via an optimization problem.

The Performance Estimation Problem (PEP)

What is PEP?

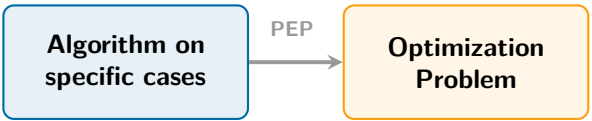
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Algorithm on specific cases

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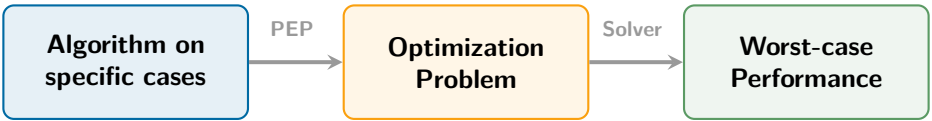
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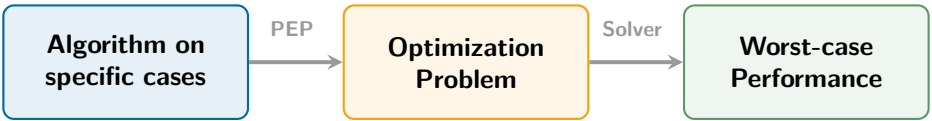
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Postdoc Goal: Adapting PEP for Control Systems

- **(Proof of Concept):** Evaluate robustness of known controllers (LQR) under structured uncertainties.
- **(Long-Term Goal):** Plenty of directions: worst-case data-driven? Robust estimation?

Proof of Concept: Robustness of LQR under LTI perturbations

Theoretical Design

- Perfect nominal model
  $x_{k+1} = Ax_k + Bu_k$
- Quadratic cost

$$\sum_{k=0}^{N-1} x_k^\top Q_K x_k + x_N^\top S x_N$$

- **Infinite** time horizon ($t \to \infty$)
- Yields optimal LQR controller K

Proof of Concept: Robustness of LQR under LTI perturbations

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$\Longrightarrow$

Real-World Deployment

- Parametric uncertainties: LTI
  $(A + \Delta_A,\ B + \Delta_B)$
- **Finite** duration operations
- Short-term transient behavior matters!

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- μ-analysis for *asymptotic* stability ($t \to \infty$).
- IQC handle finite horizons but introduce LTV.

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What is the worst-case behavior of the system subject to a norm-bounded perturbation?

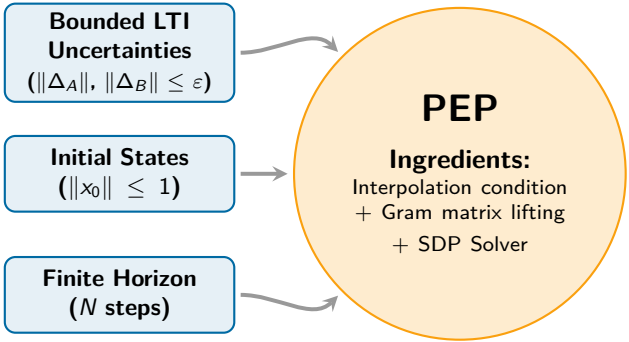
The PEP Framework Applied to Control

Bounded LTI Uncertainties
($\|\Delta_A\|, \|\Delta_B\| \leq \varepsilon$)

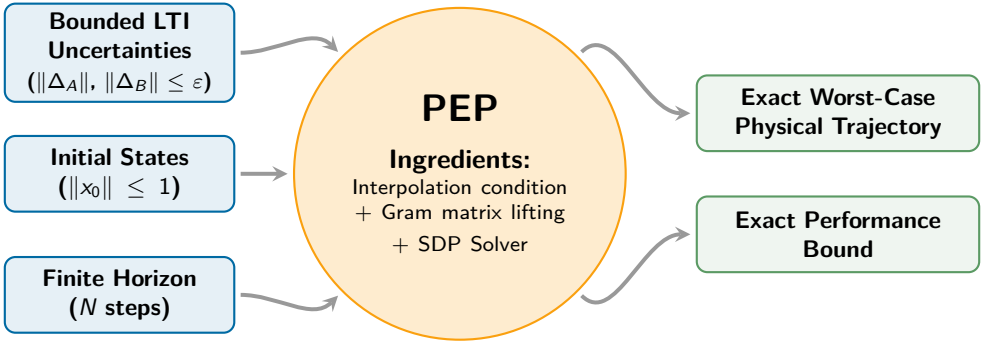
Initial States
($\|x_0\| \leq 1$)

Finite Horizon
(N steps)

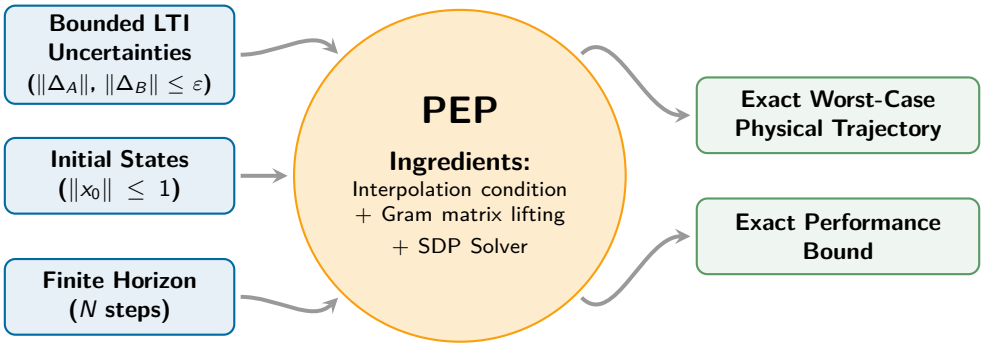
The PEP Framework Applied to Control



The PEP Framework Applied to Control



The PEP Framework Applied to Control



Interest for Control

- Guarantees **valid static LTI perturbations**.
- Extracts the **worst-case input/state sequence**.

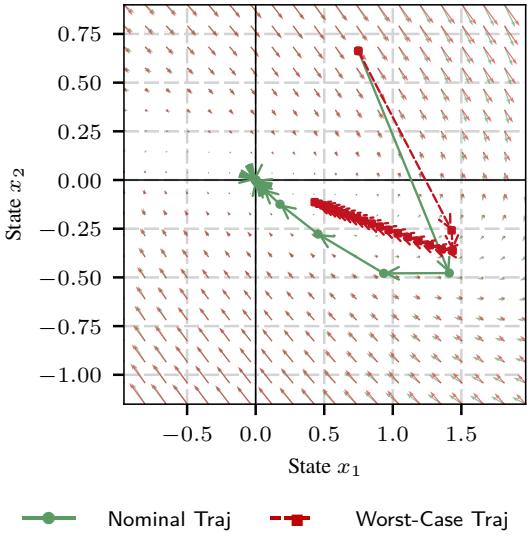
Solutions: Worst-case Trajectories

Monotonic 1D Manifold

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\|\Delta_A\| \leq 0.22, \quad \|\Delta_B\| = 0$$

$$Q = R = S = \mathrm{Id}$$
$$N = 20$$



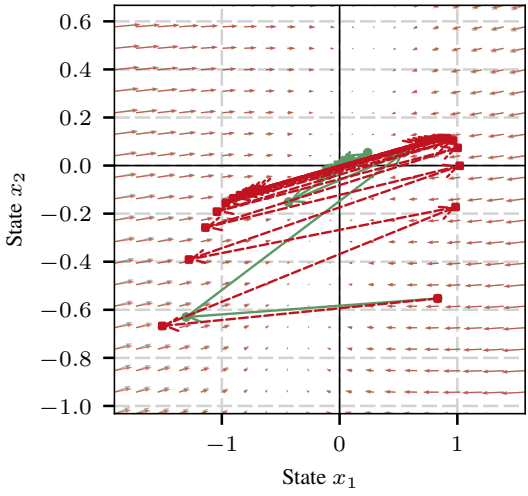
Solutions: Worst-case Trajectories

Oscillatory 1D Manifold

$$A = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\|\Delta_A\| \leq 0.2, \quad \|\Delta_B\| \leq 0.1$$

$$Q = R = S = \mathrm{Id}$$
$$N = 20$$



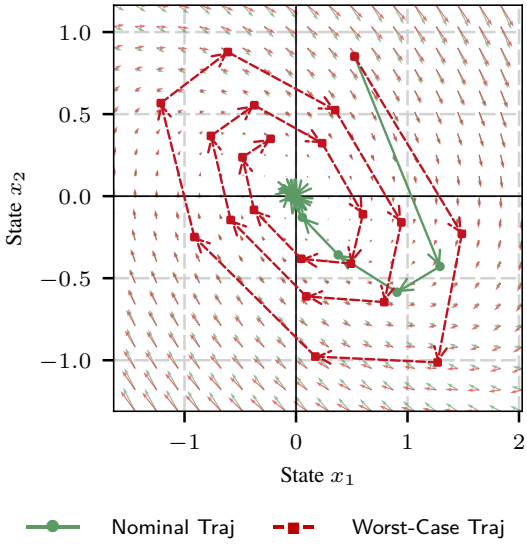
Solutions: Worst-case Trajectories

Spiraling 2D Subspace

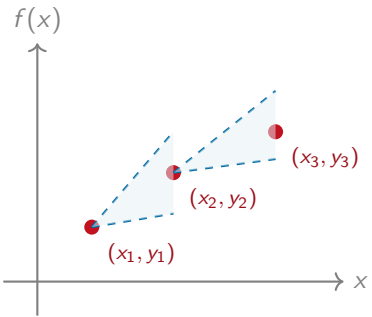
$$A = \begin{bmatrix} 1 & 0.7 \\ -0.7 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\|\Delta_A\| \leq 0.1, \quad \|\Delta_B\| \leq 0.25$$

$$Q = R = S = \mathrm{Id}$$
$$N = 20$$



The next step: System Identification via Interpolation?



Interpolation Conditions bounding the system

- **The Data:** We collect data of the system: $y_k = f(x_k)$.
- **The Rules:** Using knowledge about the physics of the system (e.g., Slotine's contractive systems, or bounded Jacobians).
- **System ID via PEP:** *"Out of all systems that match this data AND these physical rules, what are optimal controllers for them?"*

Summary & Perspectives

Ph.D. Research

Université de Lorraine

- **Tools:** Game Theory, Networks, Opinion Dynamics
- **Goal:** Modeling socio-environmental feedback loops & polarization.

Postdoctoral Research

UCLouvain

- **Tools:** PEP, Optimization, Semidefinite Programming
- **Goal:** Bring PEP to the control community.

Thank you for your attention!

I'm happy to become an Eulerian!

Altafini, Claudio (Apr. 2013). "Consensus Problems on Networks With Antagonistic Interactions". In: *IEEE Trans. Autom. Control* 58.4, pp. 935–946. ISSN: 1558-2523. DOI: 10.1109/TAC.2012.2224251.

Bizyaeva, Anastasia, Alessio Franci, and Naomi Ehrich Leonard (Mar. 2023). "Nonlinear Opinion Dynamics With Tunable Sensitivity". In: *IEEE Trans. Automat. Contr.* 68.3, pp. 1415–1430. ISSN: 0018-9286, 1558-2523, 2334-3303. DOI: 10.1109/TAC.2022.3159527.

Couthures, Anthony et al. (June 2024a). "Analysis of a Continuous Opinion and Discrete Action Model Coupled with an External Dynamics". In: *2024 Eur. Control Conf. ECC*, pp. 3392–3397. DOI: 10.23919/ECC64448.2024.10591291.

Couthures, Anthony et al. (Dec. 2024b). "Analysis of an Opinion Dynamics Model Coupled with an External Environmental Dynamics". In: *Chaos, Solitons & Fractals* 189, p. 115719. ISSN: 0960-0779. DOI: 10.1016/j.chaos.2024.115719.

Couthures, Anthony et al. (Apr. 2025). "Bifurcation Analysis of an Opinion Dynamics Model Coupled with an Environmental Dynamics". In: arXiv:2504.03419. arXiv. DOI: 10.48550/arXiv.2504.03419. arXiv: 2504.03419 [eess].

Couthures, Anthony et al. (Mar. sub2026). "International Climate Policy as a Potential Game: Strategic Incentives and Path-Dependent Dynamics". In: 6351327. DOI: 10.2139/ssrn.6351327. Social Science Research Network: 6351327.

Gray, Rebecca et al. (June 2018). "Multiagent Decision-Making Dynamics Inspired by Honeybees". In: *IEEE Trans. Control Netw. Syst.* 5.2, pp. 793–806. ISSN: 2325-5870. DOI: 10.1109/TCNS.2018.2796301.

Ito, Hiromu and Masato Yamamichi (Nov. 2024). "A Complete Classification of Evolutionary Games with Environmental Feedback". In: *PNAS Nexus* 3.11, pgae455. ISSN: 2752-6542. DOI: 10.1093/pnasnexus/pgae455.

Mroué, Bouchra et al. (2024). "A Quadratic Static Game Model for Assessing the Impact of Climate Change". In: *Complex Netw. Their Appl. XII*. Ed. by Hocine Cherifi et al. Cham: Springer Nature Switzerland, pp. 383–394. ISBN: 978-3-031-53499-7. DOI: 10.1007/978-3-031-53499-7_31.

Mroué, Bouchra et al. (June 2025). "On the Impact of Cross-Country Imitation on Climate Change: A Game-Theoretical Analysis". In: *IFAC Journal of Systems and Control* 32, p. 100309. ISSN: 2468-6018. DOI: 10.1016/j.ifacsc.2025.100309.

Tilman, Andrew R., Joshua B. Plotkin, and Erol Akçay (Feb. 2020). "Evolutionary Games with Environmental Feedbacks". In: *Nat Commun* 11.1, p. 915. ISSN: 2041-1723. DOI: 10.1038/s41467-020-14531-6.

Weitz, Joshua S. et al. (Nov. 2016). "An Oscillating Tragedy of the Commons in Replicator Dynamics with Game-Environment Feedback". In: *Proc. Natl. Acad. Sci.* 113.47, E7518–E7525. DOI: 10.1073/pnas.1604096113.