A static game model for analyzing country carbon emissions

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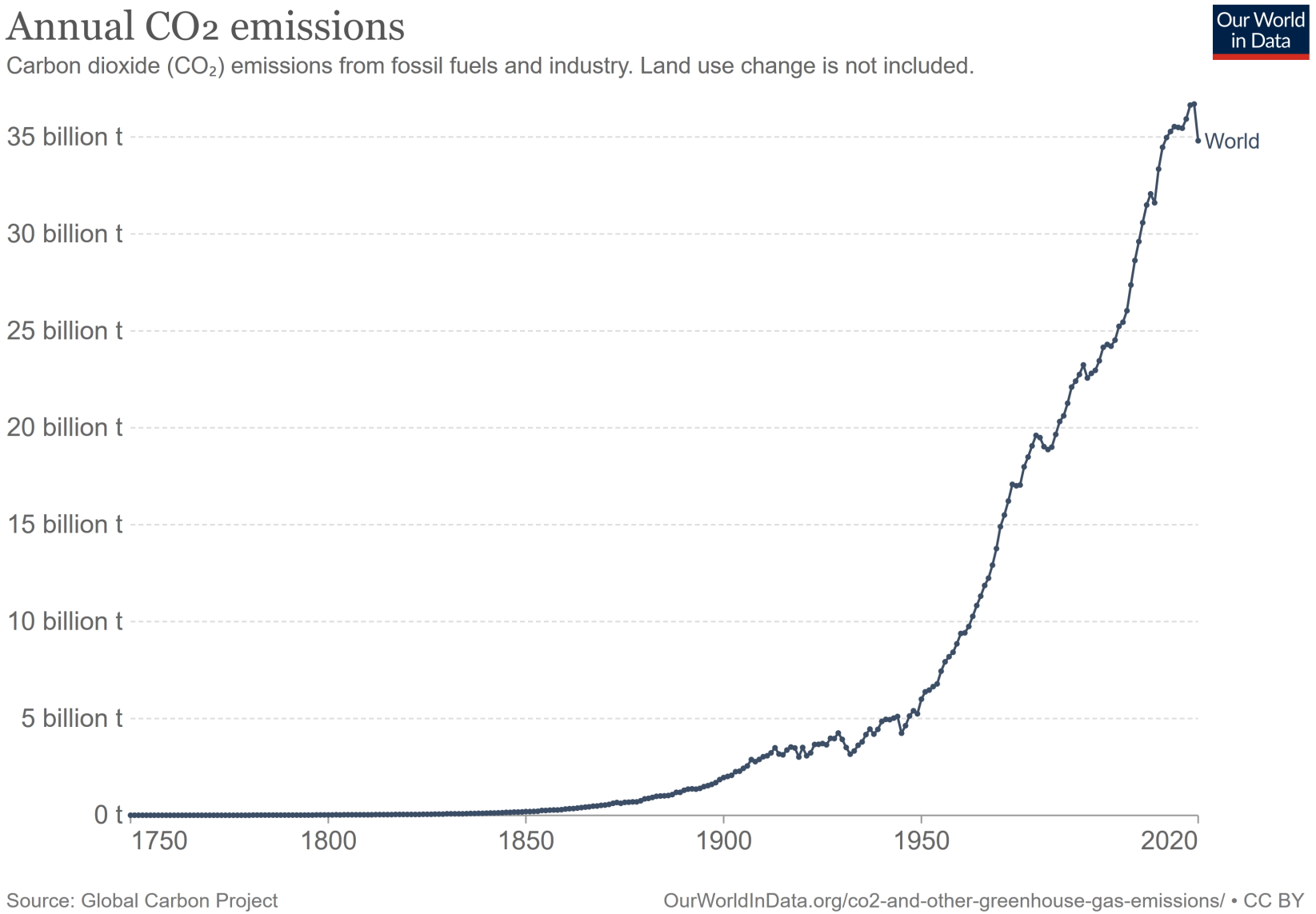
November 30, 2022



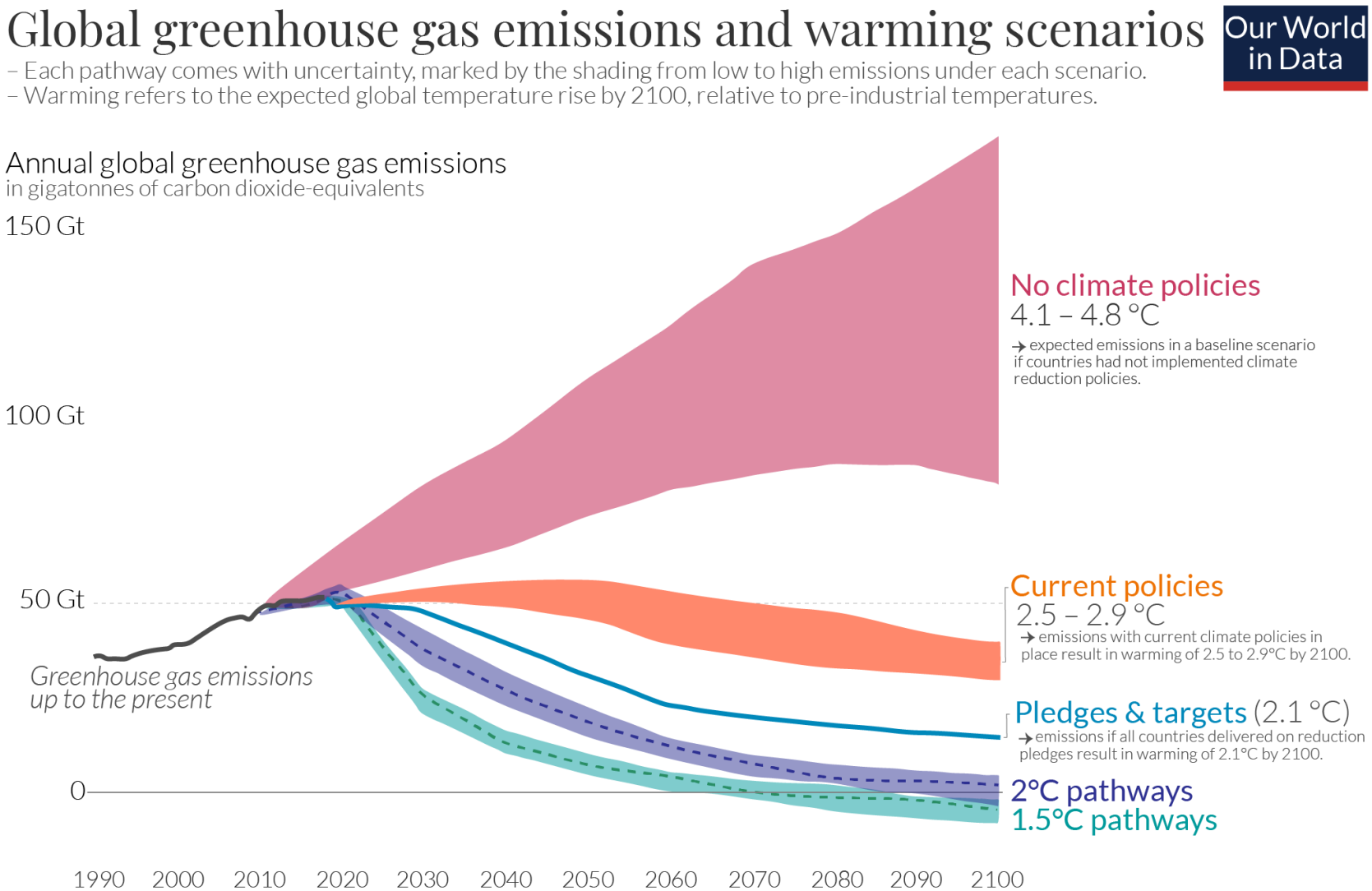




Introduction



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Actual Decarbonization Strategies

- Too much empirical and not enough modeling
- The proposed strategies lack of coordination needed IPCC recommendations

Game theory understands phenomena when agents interact

Game theory + Climate modeling
=
Decarbonization strategies
sustainable from competitive perspective

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$$\text{Game theory} + \text{Climate modeling}$$
$$=$$
$$\underline{\text{Decarbonization strategies sustainable from competitive perspective}}$$

State of the art

- Climate: Provide models and projections,
  [IPCC, 2021], [Geoffroy et al., 2013], [Meadows et al., 1972],
  [Joos et al., 2013],...
- Economy: Do control with Price of carbon,
  [Nordhaus, 2017], [Kellett et al., 2019], [Hsiang et al., 2017],
  [Rezai et al., 2020],...
- Game Theory: Hypothetical games (without simulation),
  [Chander, 2018], [Finus and Rundshagen, 2003],
  [Martimort and Sand-Zantman, 2016], [Haurie et al., 2012],...

Outline

1. General One-Shot Game
   - A model from literature
   - Our model addaptations
   - General properties of our game
   - Existence of Pure Nash Equilibrium
   - Pure Nash Equilibrium determination
2. Simple Climate Model (SCM)
   - Models of carbon cycle
   - Radiative forcing
   - Temperature Dynamic Models
3. Quadratic Case study
4. Numerical Study

Outline for section General One-Shot Game

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From literature : Finus model

- N countries, $\mathcal{N} = \{1, \ldots, N\}$
- a_n action of player n continuous
  $\Rightarrow$ CO2 player n plan to emit
- $B_n(a_n)$ benefit function for player n
- $D_n(\sum_{k=1}^{N} a_k)$ individual damage for n caused by global emissions
- Payoff function

$$u_n(a_n, \sum_{k=1}^{N} a_k) = B_n(a_n) - D_n(\sum_{k=1}^{N} a_k)$$

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Lack of Finus model

Issues with Finus's Model:

- Too generic
- No equilibrium existence guarantee
- No equilibrium determination
- Without a climate modeling
- Not state dependent
- No simulation possible
- Strong assumptions

Game theory framework

- a_n action of player n $a_n \in [0, e_n^{max}] = \mathcal{A}_n$, $\mathcal{A} = \prod_{n \in \mathcal{N}} \mathcal{A}_n$
  $\Rightarrow$ CO2 player n plan to emit
- $B_n(a_n)$ benefit function for player n
- $D(y_{AT}(\sum_{k=1}^{N} a_k; x))$ damage caused by global emissions globally
- $y_{AT} : \sum_{n \in \mathcal{N}} \mathcal{A}_n \to \mathbb{R}$ mean global atmospheric temperature
- x state, describing the climate
- $\delta_n \geq 0$ multiplier of damage to n.
- Payoff function

$$u_n \left(a_n, \sum_{k=1}^{N} a_k; x \right) = B_n(a_n) - \delta_n D \left(y_{AT} \left(\sum_{k=1}^{N} a_k; x \right) \right)$$

Finus Assumptions

Assumptions $\forall n \in \{1, \ldots, N\}$,

- No benefit without emissions $\Rightarrow B_n(0) = 0$
  Benefit :

$$\left.\begin{array}{l}\text{Non decreasing}\\ \text{Concave}\end{array}\right\} \Longrightarrow \left\{\begin{array}{ll} B_n' \geq 0, & \forall 0 \leq a_n \leq e_n^{\max},\\ B_n'' \leq 0, & \forall a_n \geq 0.\end{array}\right.$$

- No damage without emissions $\Rightarrow D_n(0) = 0$
  Damage :

$$\left.\begin{array}{l}\text{Non decreasing}\\ \text{Convex}\end{array}\right\} \Longrightarrow \forall \sum_{k=1}^{N} a_k \geq 0, \left\{\begin{array}{l} D_n' \geq 0\\ D_n'' \geq 0\end{array}\right.$$

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Relaxation and change in assumptions

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- ~~No damage without emissions $\Rightarrow D(0) = 0$~~
  Damage :

$$\left.\begin{array}{l}\text{Non decreasing w.r.t } y_{AT} \\ \text{Convex w.r.t } y_{AT}\end{array}\right\} \Longrightarrow \forall y_{AT}, \left\{\begin{array}{l} D' \geq 0 \\ D'' \geq 0\end{array}\right.$$

General properties of our game

We recall,

Definition: Weighted potential game

$\Gamma = (\mathcal{N}, \mathcal{A}, (u_n)_{n\in\mathcal{N}})$ is a weighted potential game iff $\exists$ a potential function $\phi : \mathcal{A} \to \mathbb{R}$ s.t, $\forall n \in \mathcal{N}$:

$$u_n(a_n, a_{-n}) - u_n(a'_n, a_{-n}) = w_n(\phi(a_n, a_{-n}) - \phi(a'_n, a_{-n})),$$

where $(w_n)_{n\in\mathcal{N}}$ positive numbers i.e. weights.

General properties of our game

Proposition 1

The game Γ is a weighted potential game with the following potential, $\forall a \in \mathcal{A}$, $\forall x$,

$$\phi(a;x) = \sum_{k=1}^{N} \frac{1}{\delta_n} B_n(a_n) - D(y_{AT}(\sum_{k=1}^{N} a_k; x))$$

Lemma 1: [Monderer and Shapley, 1996]

ϕ potential function of weighted potential game $\Gamma = (\mathcal{N}, \mathcal{A}, (u_n))$, then the set of Nash equilibria $\mathbb{NE}(\Gamma)$ of the game Γ coincides with the set of equilibrium of $\hat{\Gamma} = (\mathcal{N}, \mathcal{A}, (\phi))$. i.e

$$\mathbb{NE}(\Gamma) = \mathbb{NE}(\hat{\Gamma}).$$

Existence of Pure Nash Equilibrium

Proposition 2

If the potential ϕ admits at least one maximum over the set $\mathcal{A}$, then the game Γ has at least one pure Nash equilibrium.

Proposition 3

Assuming the functions B_n, for all $n \in \mathcal{N}$, D and y_{AT} being continuous functions with respect to $a \in \mathcal{A}$, then Γ admits at least one pure Nash equilibrium.

Pure Nash Equilibrium determination: ϕ strict convex

Proposition 4

If ϕ is strictly convex, under the assumption above, $\phi : \mathcal{A} \to \mathbb{R}$ has at most 2^N maximum. Moreover, the set of Nash equilibria of Γ is given by

$$\mathbb{NE}_\Gamma = \{(a_1, \ldots, a_N) \mid a_n \in \{0, e_n^{max}\} \quad n \in \mathcal{N}\}.$$

If players assume convex Benefit, they will either pollute as much as they can or completely stop!

Pure Nash Equilibrium determination: ϕ strict concave

If, ϕ strictly concave $\Rightarrow$ uniqueness of a^*. A solution of

$$\text{Maximize } \phi(a) \text{ s.t } a \in \mathcal{A}, \quad \forall n \in \mathcal{N}.$$

$\mathcal{A}$ is convex $\Rightarrow$ we can find KKT conditions for $a^* = (a_1^*, \ldots, a_N^*)$
$\underline{\lambda}_n$ and $\overline{\lambda}_n$ the conditions over $a_n^* \geq 0$ and $a_n^* \leq e_n^{max}$

$$KKT = \begin{cases} \partial_{a_n}\phi(a^*) = \overline{\lambda}_n - \underline{\lambda}_n, \ \forall n \in \mathcal{N} \\ \underline{\lambda}_n a_n^* = 0, \ \forall n \in \mathcal{N} \\ \overline{\lambda}_n (a_n^* - e_n^{max}) = 0, \ \forall n \in \mathcal{N} \\ \overline{\lambda}_n, \underline{\lambda}_n \geq 0, \ \forall n \in \mathcal{N}. \end{cases}$$

a^* being an equilibrium if solution of the system with N equations:

$$\left\{ \frac{1}{\delta_n} \frac{\partial B_n}{\partial a_n}(a_n^*) - \left[y'_{AT} D'(y_{AT}) \right](a^*; x) = \overline{\lambda}_n - \underline{\lambda}_n, \quad \forall n \in \mathcal{N} \right.$$

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Outline for section Simple Climate Model (SCM)

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Linear models of carbon cycle

m different boxes $\Rightarrow$ m different equations

$$z_{AT}(t) = b_z^{\mathrm{T}} z(t),$$

$$z(t) = \mathbf{A}_z z(t-1) + d_z E(t) = \mathbf{A}_z^t z(0) + \sum_{s=1}^{t} \mathbf{A}_z^{t-s} d_z E(s),$$

where

- $E(t)$ is the global CO2 emission at time t,
- $z(t) \Rightarrow m$-vector of amount of carbon in boxes at t
- $A_z \Rightarrow$ flows between boxes
- $d_z \Rightarrow$ CO2 emissions $\rightarrow$ boxes

Linear models of carbon cycle

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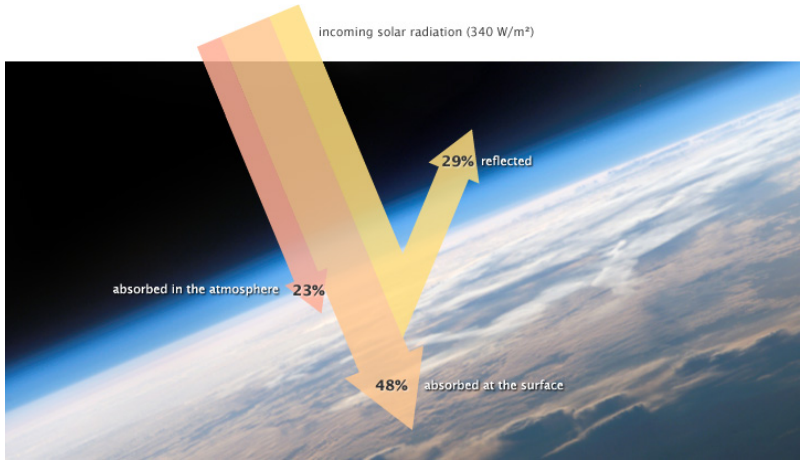
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Radiative forcing



$$F(t) = F_{2\times CO2} \log_2 \left(\frac{z_{AT}(t)}{z_{AT}(1750)} \right) + F_{EX}(t), \quad \forall t \in \mathbb{R}^+$$

- $F_{2\times CO2}$ coefficient for radiative forcing for a doubled CO2 atmospheric concentration
- $z_{AT}(1750)$ reference concentration from pre-industrial era
- $F_{EX}(t)$ is an exogenous forcing from other gases

Linear models of temperature dynamic

m' different boxes $\Rightarrow$ m' different equations

$$y_{AT}(t) = b_y^{\mathrm{T}} y(t),$$

$$y(t) = \mathbf{A}_y y(t-1) + d_y F(t) = \mathbf{A}_y^t y(0) + \sum_{s=1}^{t} \mathbf{A}_y^{t-s} d_y F(s),$$

where

- $F(t)$ is the global radiative forcing at time t
- $y(t) \Rightarrow m'$-vector of temperature in boxes at t
- $A_y \Rightarrow$ flows between boxes
- $d_y \Rightarrow$ radiative forcing $\rightarrow$ boxes
- $b_y \Rightarrow$ boxes $\rightarrow$ atmospheric temperature

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Benefit and Damage functions quadratic

Benefit functions quadratic in action

$$B_n(a_n) = \sum_{i=0}^{2} \beta_{i,n} a_n^i,$$

Damage function quadratic in temperature y_{AT}

$$D\left(y_{AT}\left(\sum a_k; x\right)\right) = \sum_{i=1}^{2} \gamma_i \left[y_{AT}\left(\sum a_k; x\right)\right]^i$$

An exact formulation in strictly concave case

Proposition

In quadratic case, if ϕ is strictly concave, then the NE of $\Gamma = (\mathcal{N}, \mathcal{A}, \mathcal{U})$ is unique and given by $a^* = (a_1^*, \ldots, a_N^*)$, where

$$a_n^* = \frac{\delta_n}{2\beta_{2,n}} \frac{L\left[G + H \log_2\left[\frac{a\xi+p}{q}\right]\right]}{a\xi + p} - \frac{\beta_{1,n}}{2\beta_{2,n}}, \quad \forall n \in \mathcal{N}$$

where ξ is the unique solution of the equation

$$a\xi^2 + b\xi + c = d \log_2\left[\frac{a\xi + p}{q}\right]$$

with a, b, c, d, p, q, G, H and L refer to parameters of the model, in the table next slide.

Exact doesn't mean easy to write!

a	$b_z^{\mathrm{T}} d_z$
b	$p + b_z^{\mathrm{T}} d_z \sum_{n \in \mathcal{N}} \frac{\beta_{1,n}}{2\beta_{2,n}}$
c	$p \sum_{n \in \mathcal{N}} \frac{\beta_{1,n}}{2\beta_{2,n}} - \frac{F_{2CO2} b_y^{\mathrm{T}} d_y a}{\ln 2} G \sum_{n \in \mathcal{N}} \frac{\delta_n}{2\beta_{2,n}}$
d	$\frac{2\gamma_2 F_{2CO2}^2 b_y^{\mathrm{T}} d_y \left[b_z^{\mathrm{T}} d_z\right]^2}{\ln 2} \sum_{n \in \mathcal{N}} \frac{\delta_n}{2\beta_{2,n}}$
p	$b_z^{\mathrm{T}} A_z z$
q	$z_{AT}(0)$
G	$\gamma_1 + 2\gamma_2 b_y^{\mathrm{T}} A_y y + 2\gamma_2 F_{2CO2} b_y^{\mathrm{T}} d_y F_{EX}$
H	$2\gamma_2 F_{2CO2} b_y^{\mathrm{T}} d_y$
L	$\frac{F_{2CO2} b_y^{\mathrm{T}} d_y a}{\ln 2}$

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What we will look at

We repeat the One-Shot game without allowing player to plan:
$\Rightarrow$ at time t the play the game with state x_t
$\Rightarrow$ the actions provide a x_{t+1}
With $t_0 = 2020,\ T = 2100$.

On the following plots,

- "——" : Nash Equilibrium strategy
- "- - -" : Social optimal strategy i.e Maximum of the sum of utilities $\sum u_k$ for one game

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Description of players and SCM

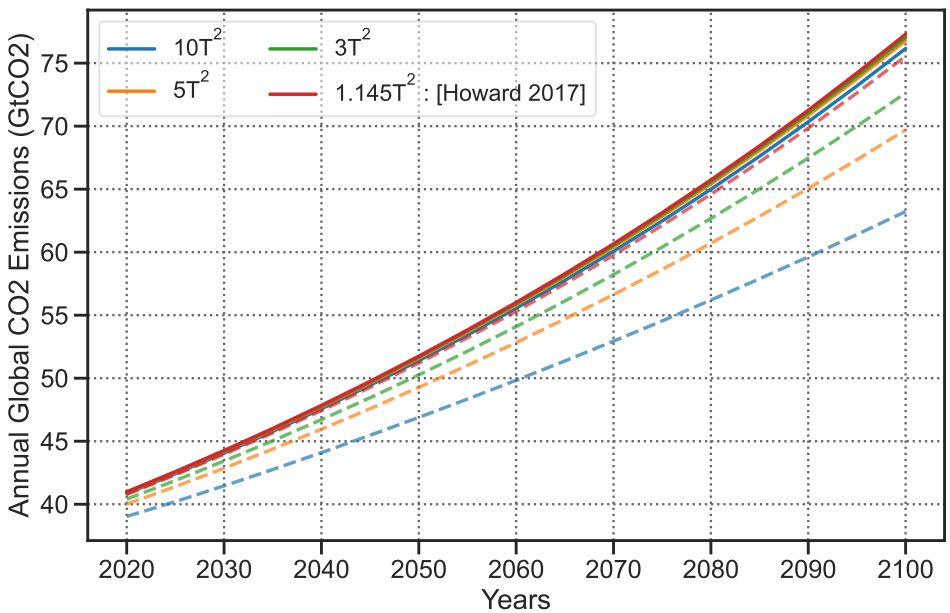
In our simulations, we will have 6 players with the following characteristics,

Player n	$e_n^{max}(0)$	$GDP_n^{max}(0)$	δ_n	Close Existing Entity	Update
Player 1	11	15	1.1847	China	yes
Player 2	7	18	1.1941	USA	yes
Player 3	4	17	1.1248	European Union	no
Player 4	3	2.6	0.9074	India	yes
Player 5	2	1.6	1.2866	Russia	yes
Player 6	5	26	1.1847	Asia $\setminus$ China, India	yes

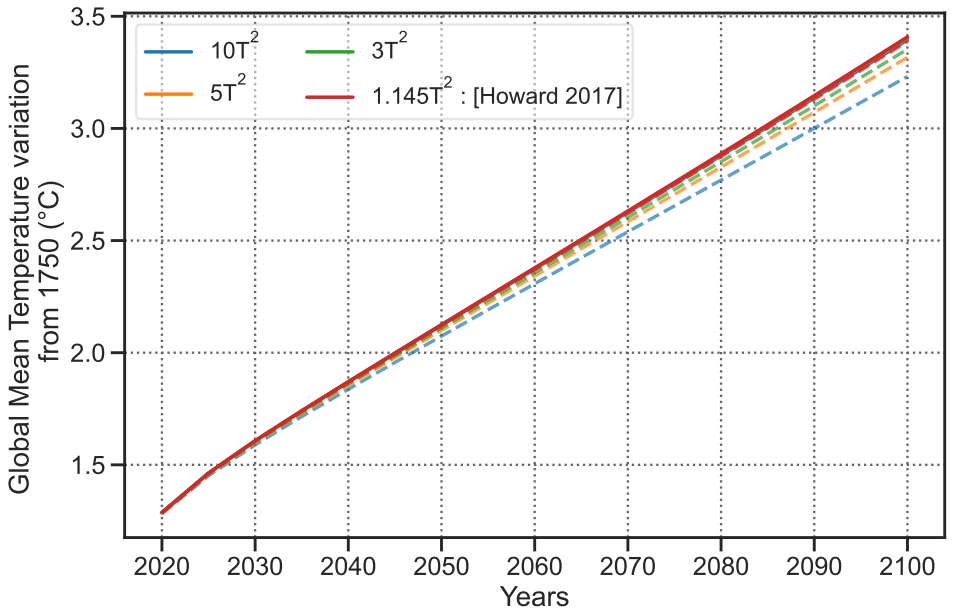
SCM = Carbon Cycle [Joos et al., 2013] + Temperature Dynamic [Geoffroy et al., 2013].

Damage doesn't change much behaviors

Players have concave quadratic benefit functions with max at e_n^{max}



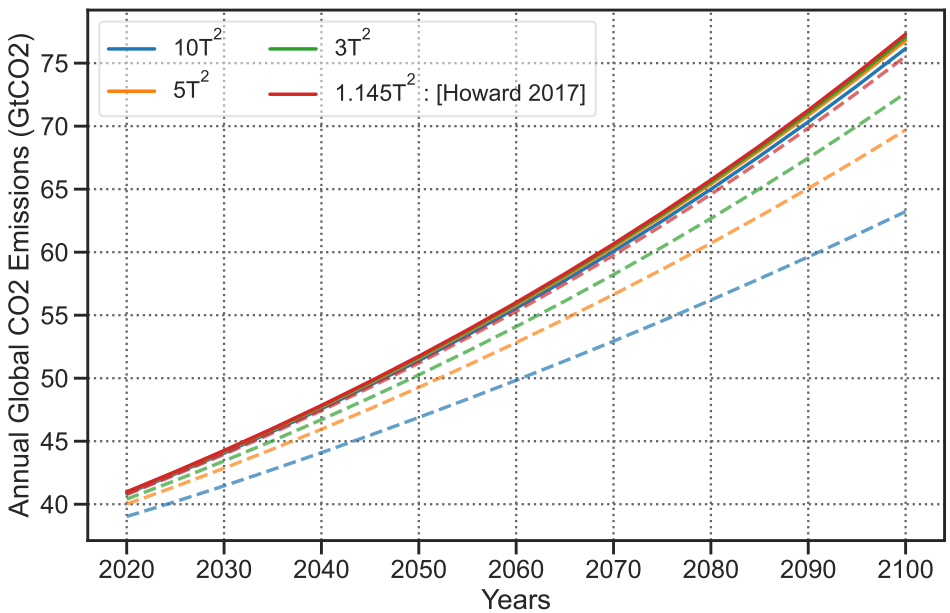
(a) Annual Global CO2 Emissions w.r.t different damage functions



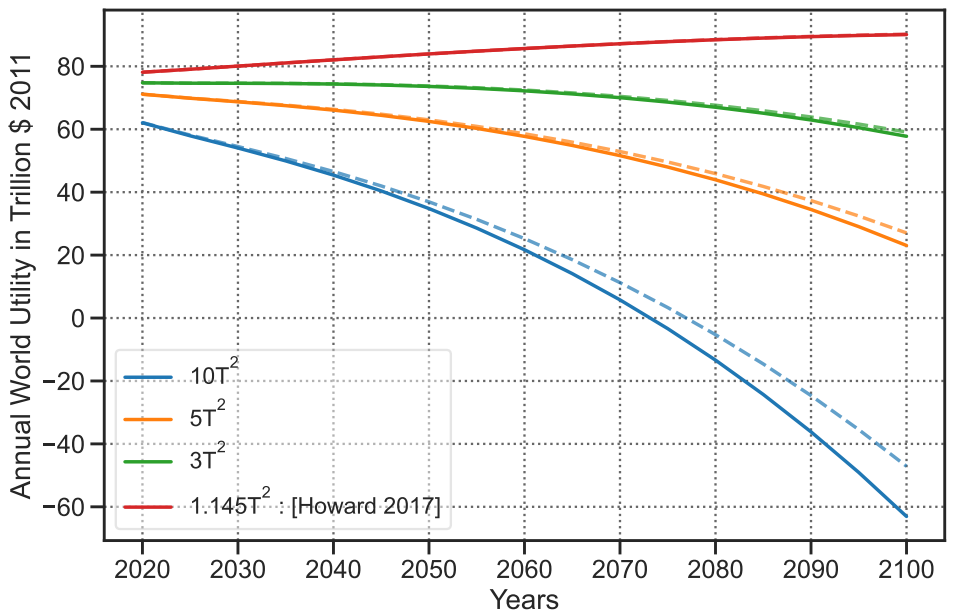
(b) Global Mean Temperature variation w.r.t different damage functions

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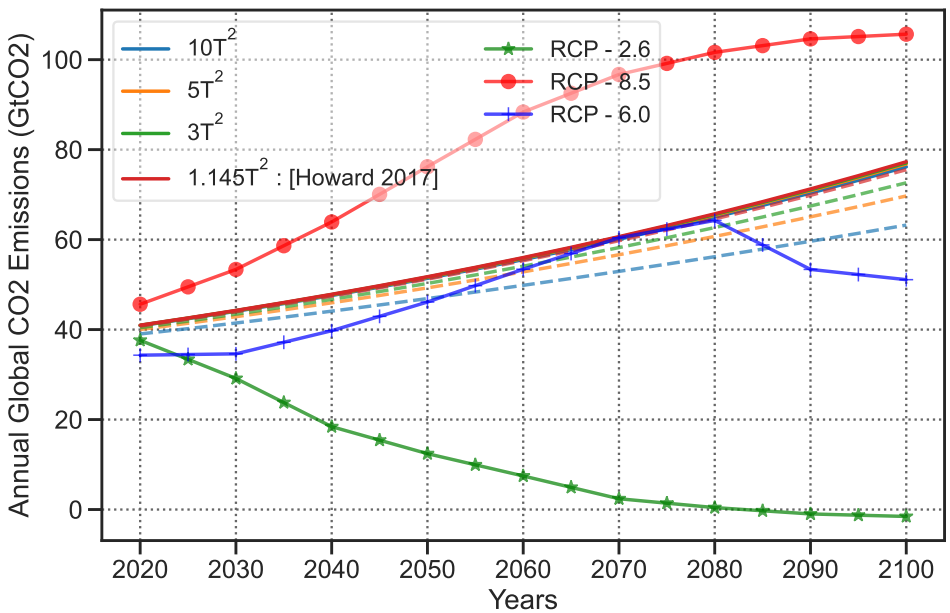
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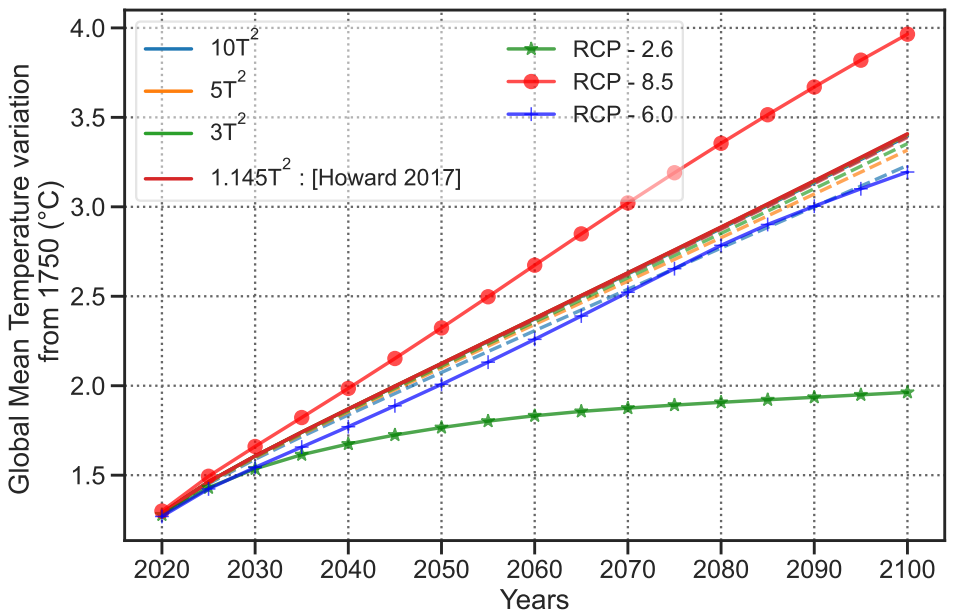
(b) Annual World Utility w.r.t different damage functions

Damage doesn't change much behaviors

Players have concave quadratic benefit functions with max at e_n^{max}



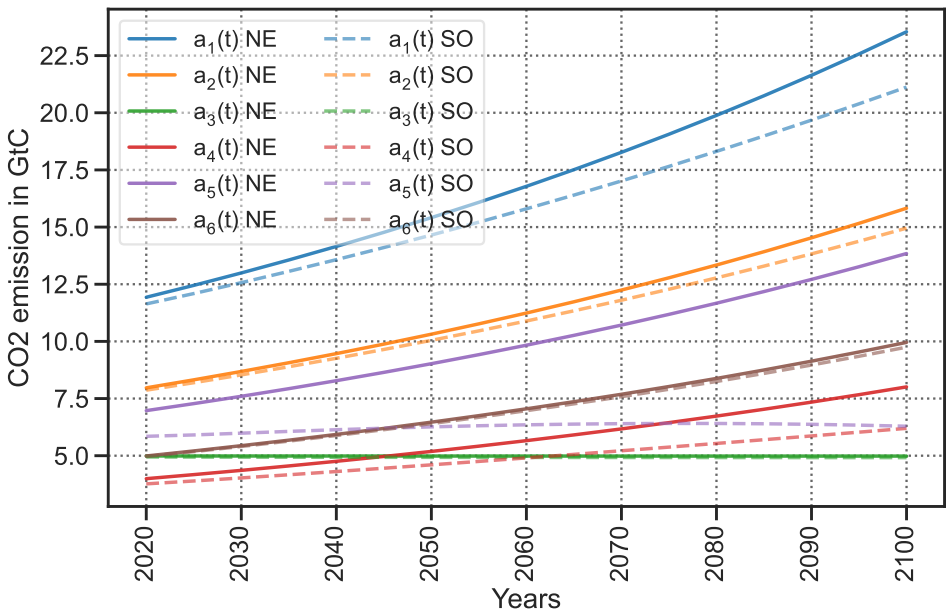
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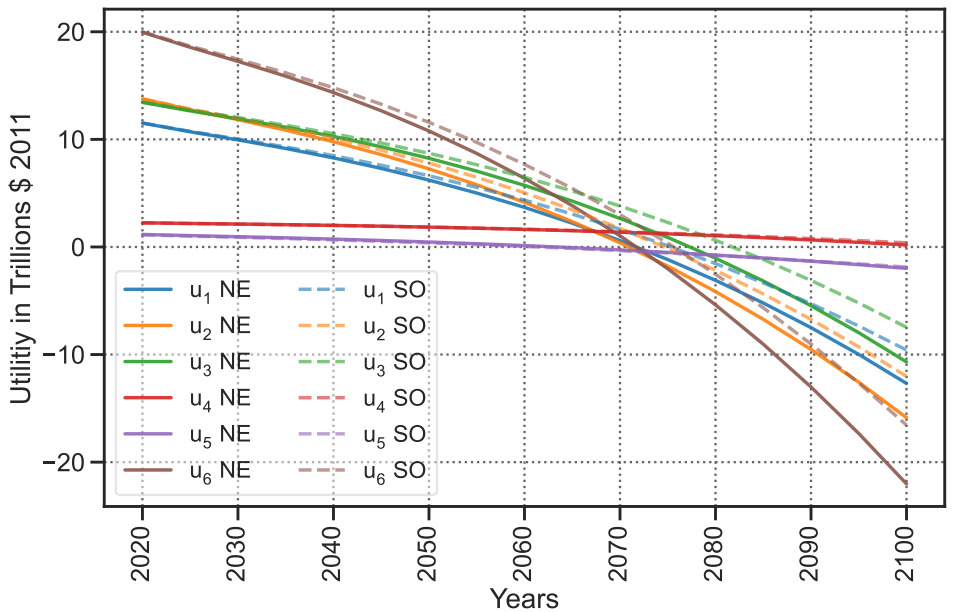
(b) Global Mean Temperature variation w.r.t different damage functions

What happen for the players

Concave benefit and Damage are $10\,T^2$.



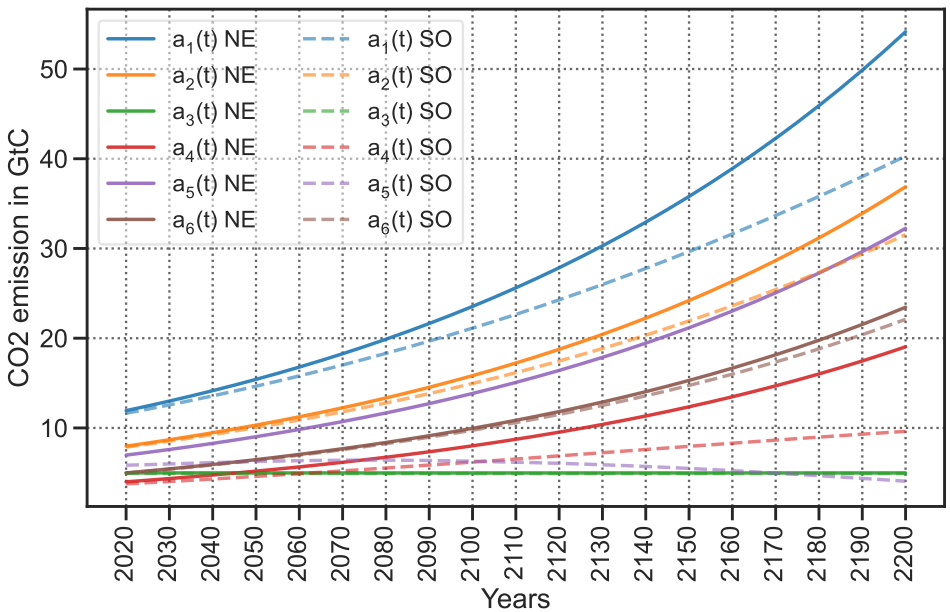
(a) Annual Individual CO2 Emissions w.r.t damage functions $10\,T^2$



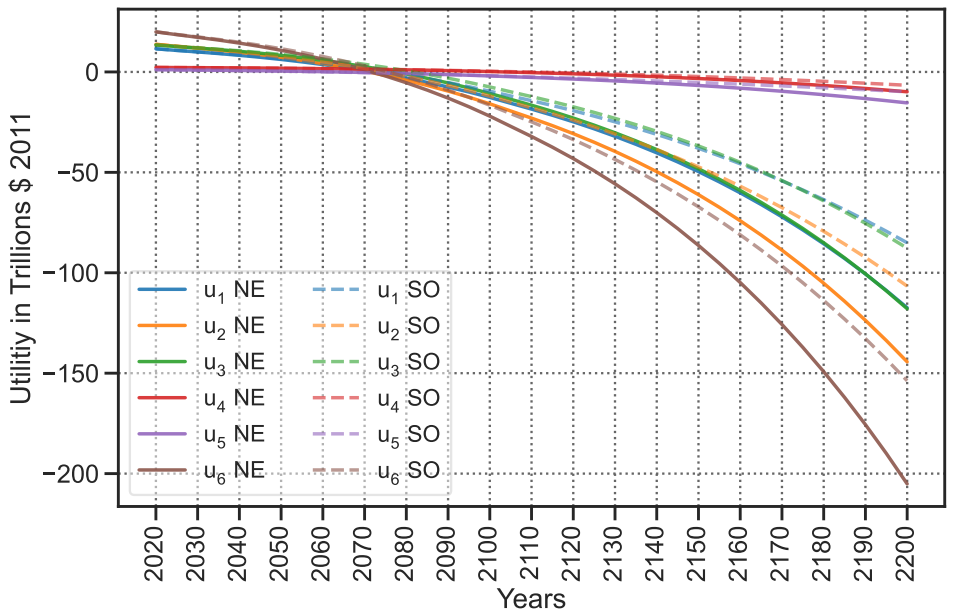
(b) Annual Individual Utilities w.r.t damage functions $10\,T^2$

Does the Horizon change something ?

Concave benefit and Damage are $10\,T^2$.



(a) Annual Individual CO2 Emissions w.r.t damage functions $10\,T^2$



(b) Annual Individual Utilities w.r.t damage functions $10\,T^2$

Does we find back the Convex case NE ?

If the Benefit are convex, ϕ is often convex. From the mathematical analysis we found that

Proposition 4

If ϕ is strictly convex, under the assumption above, $\phi : \mathcal{A} \to \mathbb{R}$ has at most 2^N maximum. Moreover, the set of Nash equilibria of Γ is given by

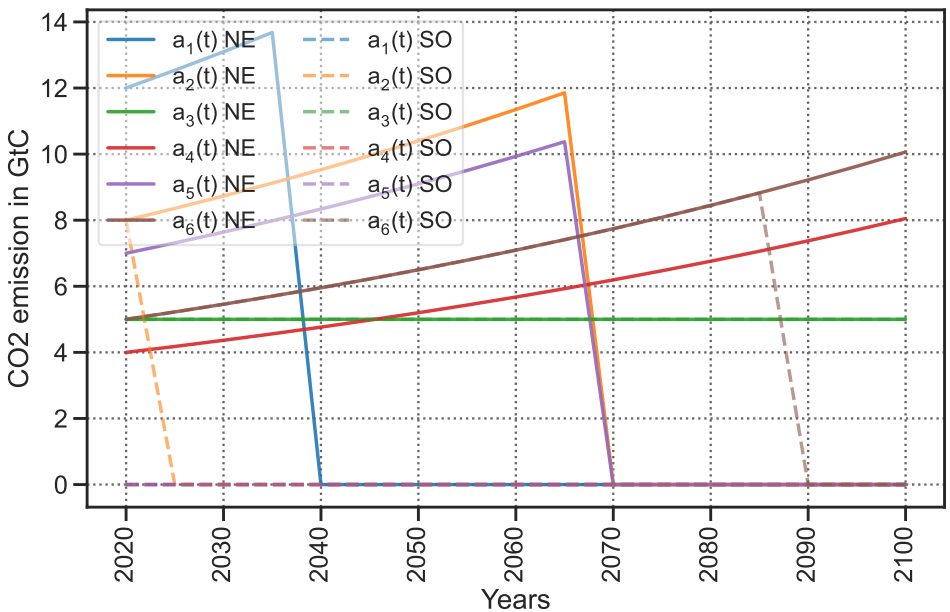
$$\mathbb{NE}_\Gamma = \{(a_1, \ldots, a_N) \mid a_n \in \{0, e_n^{max}\} \quad n \in \mathcal{N}\}.$$

Do we observe it ?

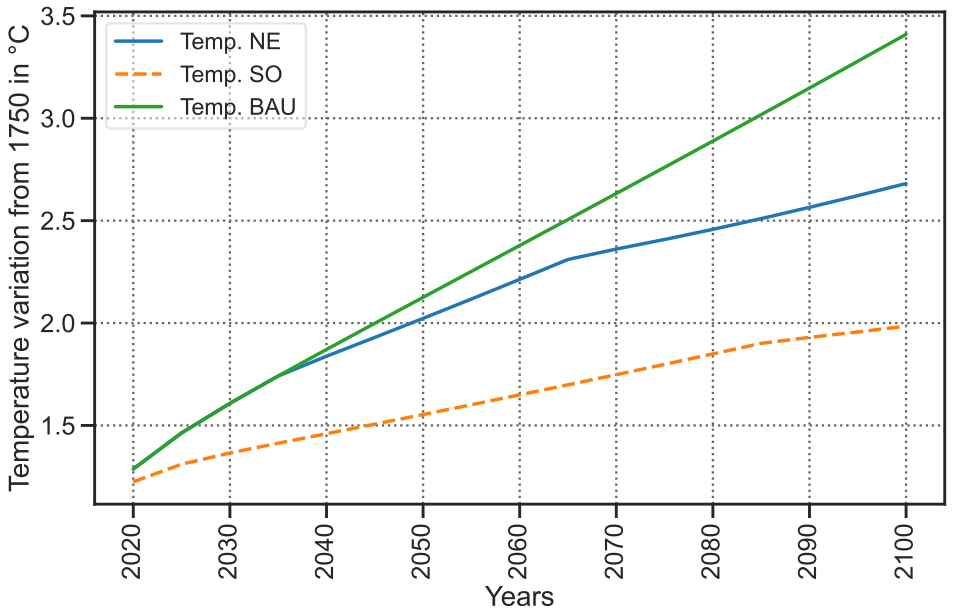
Does we find back the Convex case NE ?

Convex benefit and Damage are $600\,T^2$ (very high for make it appear before 2100)

$\Rightarrow$ YES there is a shift from $e_n^{max}(t)$ to 0 in one turn.



(a) Annual Individual CO2 Emissions w.r.t damage function $600\,T^2$



(b) Global Mean Temperature variations w.r.t damage function $600\,T^2$

Conclusion

- Contributions
  - New structured model for experimenting
  - Equilibrium analysis: existence and determination
  - Realistic Simulator: allowing us to see behaviors of players and impacts on climate, soon available on Github

- Future research topics
  - Improve our One-Shot game: fitting more to the reality, state dependent benefit function, allowing player to invest in renewable, fixing a maximum amount of fossil energy, use carbon pricing,...
  - Dynamic game model: strategies, add damages with rate of increase in temperature,...
  - Different types of players: the ones already shifting to non fossil energy and the others

Summary of presentation

Thanks for your attention!

In this talk, we presented

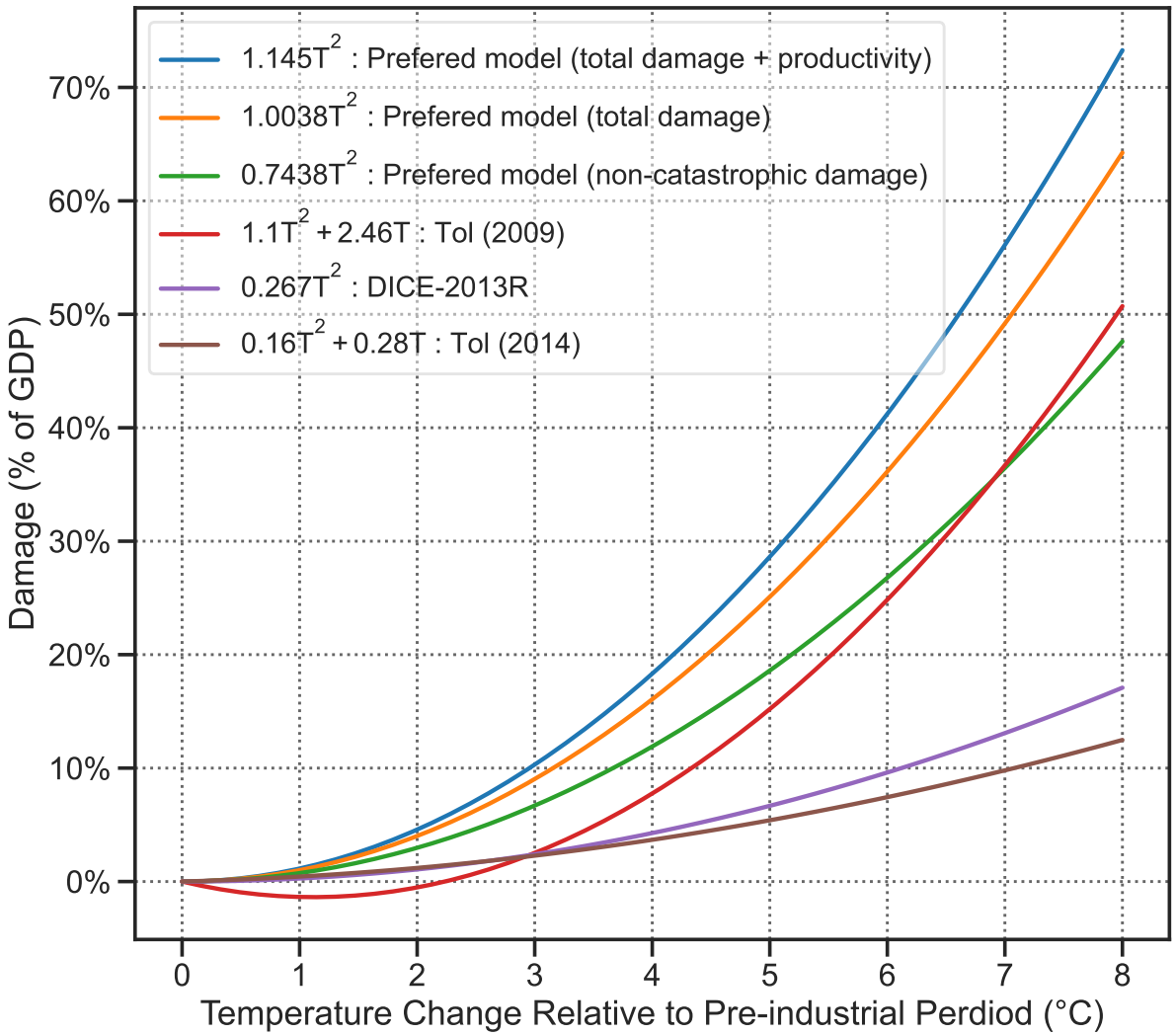
- A model of game from literature: Finus Model
- An improved version of this game taking into account climate
- An equilibrium analysis of this new game
- Models economists use for modeling the climate
- An explicit formulation of the Nash equilibrium in the quadratic concave case
- Simulations of the "repeated" One-Shot game

Utilities of the One-Shot Game at time t

$$u_n\left(a_n(t), \sum_{k=1}^{N} a_k(t); x_t\right) = B_n(a_n(t)) - \delta_n D\left(y_{AT}\left(\sum_{k=1}^{N} a_k(t); x_t\right)\right)$$

- $a_n(t) \in \mathcal{A}_n(t) \Rightarrow$ action of player n at t
- $\sum_{k=1}^{N} a_k(t) \in \mathbb{A} \Rightarrow$ the aggregated actions at t
- $x_t \Rightarrow$ state at t, the couple (y_t, z_t)
- $B_n : \mathcal{A}_n \to \mathbb{R} \Rightarrow$ benefit function of n
- $D : \mathbb{R} \to \mathbb{R} \Rightarrow$ damage function all the players
- $y_{AT} : \mathbb{A} \to \mathbb{R} \Rightarrow$ mean global atmospheric temperature
- $\delta_n \Rightarrow$ impact factor of damage to n.

Polynomial Damage function of literature



Recall of our Utilities

$$u_n\left(a_n, \sum_{k=1}^{N} a_k; x\right) = B_n(a_n) - \delta_n D\left(y_{AT}\left(\sum_{k=1}^{N} a_k; x\right)\right)$$

- $a_n \in \mathcal{A}_n \Rightarrow$ action of player n
- $\sum_{k=1}^{N} a_k \in \mathbb{A} \Rightarrow$ the aggregated actions
- $x \Rightarrow$ state, i.e. a couple (y, z)
- $B_n : \mathcal{A}_n \to \mathbb{R} \Rightarrow$ benefit function of n
- $D : \mathbb{R} \to \mathbb{R} \Rightarrow$ damage function all the players
- $y_{AT} : \mathbb{A} \to \mathbb{R} \Rightarrow$ mean global atmospheric temperature
- $\delta_n \Rightarrow$ impact factor of damage to n.

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