A Planning Game for Global Decarbonization Strategies

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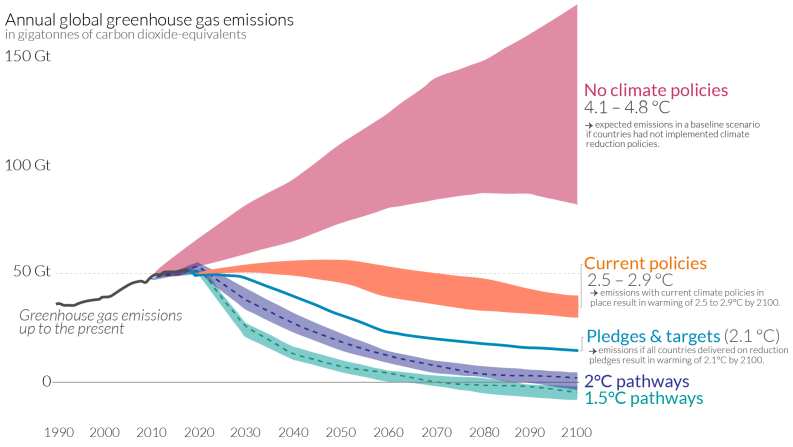


Introduction

Global greenhouse gas emissions and warming scenarios

Our World in Data

– Each pathway comes with uncertainty, marked by the shading from low to high emissions under each scenario.
– Warming refers to the expected global temperature rise by 2100, relative to pre-industrial temperatures.



Data source: Climate Action Tracker (based on national policies and pledges as of November 2021).
OurWorldinData.org – Research and data to make progress against the world's largest problems.

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Rethinking Decarbonization Strategies

Naive question at hand:

- Why governments do not utilize IPPC's more effective decarbonization trajectories?

Current observations:

- Prevalent strategies are empirically driven with insufficient modeling.
- Lack of coordination between countries within proposed strategies; does not align with IPCC recommendations.

Rethinking Decarbonization Strategies

Proposed approach to answer:

Our Perspective

Applying game theory to understand the interplay between countries in climate action.

$$\underline{\text{Game Theory}} + \underline{\text{Climate Modeling}} = \textbf{Sustainable Decarbonization Strategies}$$
$$\text{that are } \underline{\text{competitive}} \text{ and } \underline{\text{cooperative}}?$$

Our Focus:

- To present a game-theoretic approach to assess the lack of coordination and build decarbonization strategies that are sustainable.

State of the Art in Decarbonization Efforts

Climate Science

Models and projections

[IPCC, 2021]
[Geoffroy et al., 2013]
[Smith et al., 2018]
[Leach et al., 2021]
[Joos et al., 2013]
...

Economics

General equilibrium

Carbon pricing

[Nordhaus and Yang, 1996]
[Nordhaus, 2019]
[Kellett et al., 2019]
[Hänsel et al., 2020]
[Weitzman, 2010]
...

Game Theory

Prisoner Dilemma without simulation

Coalitions

[Finus, 2002]
[Chander, 2018]
[Germain et al., 2010]
[Babonneau et al., 2013]
...

Some do links:

[Dietz et al., 2021], [Tulkens and Eyckmans, 1999], [Tulkens, 2019], [Faulwasser et al., 2018],...

Observation: While multiple disciplines address decarbonization, there is still a crucial lack of inter-disciplinary coordination.

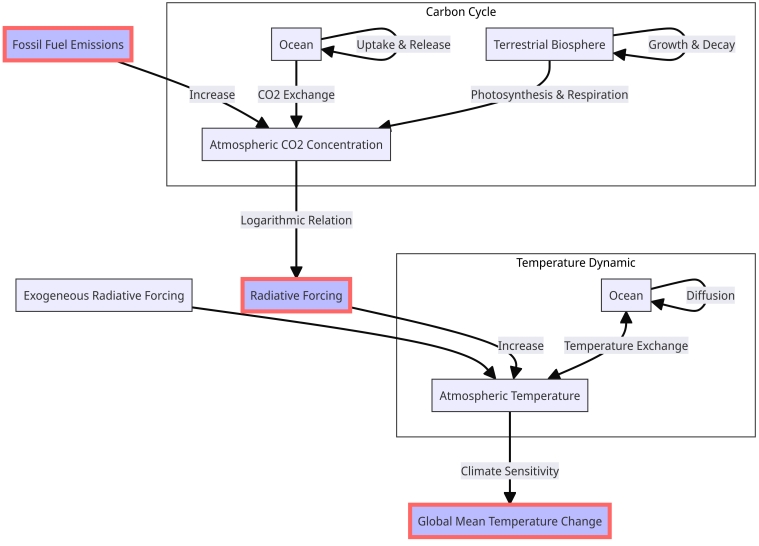
Outline

1. Models
   Climate Model (CM)
   Integrated Assessment Models (IAM)

2. General Game

3. Numerical Study

Climate Model (CM)



Simplifying the Climate Model

We use linear systems, but the approach can be generalized.

Carbon Cycle: state C

$$\begin{cases} C_{t+1} = \mathbf{A}_C C_t + b_C E_t \\ C_t^{\mathrm{AT}} = d_C^\top C_t \end{cases}$$

Temperature Dynamic: state θ

$$\begin{cases} \theta_{t+1} = \mathbf{A}_\theta \theta_t + b_\theta F_{t+1} \\ \theta_t^{\mathrm{AT}} = d_\theta^\top \theta_t. \end{cases}$$

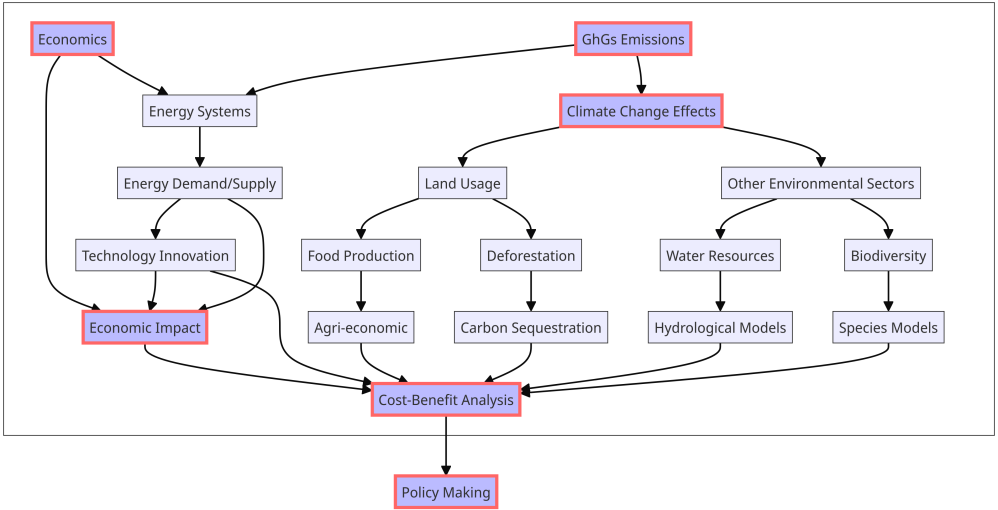
N.B: "AT"stands for atmospheric. From 2020 to 2100, with $\Delta t = 5$: $T = 16$ steps.

Logarithm relation:

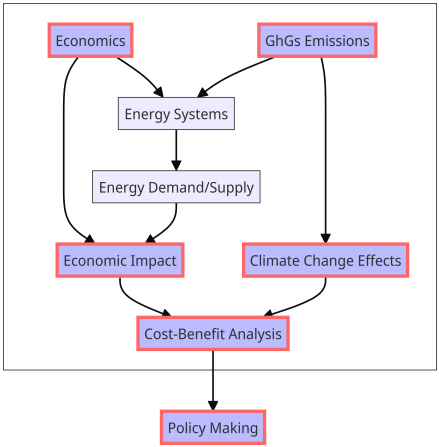
$$F_t = F_{2\times \mathrm{CO}_2} \log_2 \left(\frac{C_t^{\mathrm{AT}}}{C_{\mathrm{ref}}^{\mathrm{AT}}} \right) + F_{t,nonCO_2}.$$

Caution: Even if the systems separately are linear the climate dynamic is nonlinear due to log.

Integrated Assessment Models (IAMs)



Simplifying the IAM



Key Takeaway: Links Benefits from energy consumption and Damages from the climate.

Outline

1. Models

2. General Game
   - A Model from Literature
   - Our model adaptations
   - General properties of our game
   - Existence of Pure Nash Equilibrium
   - Pure Nash Equilibrium determination

3. Numerical Study

From Literature: The Basic Game Model

Simplifying to a Basic Game

IAMs are complex and often opaque, we return to a fundamental **one shot** game model from the literature[a].

[a] [Finus, 2002],[Tulkens, 2019],[Chander, 2018]

- Game with N countries, country n's action a_n are CO_2 emissions.
- Payoff for n is a function of its own action and the sum of all:

$$u_n(a_n, a_{-n}) = B_n(a_n) - D_n\left(\sum_{k=1}^{N} a_k\right),$$

- $B_n(a_n)$: Economic benefits country n from its emissions.
- $D_n\left(\sum_{k=1}^{N} a_k\right)$: Damages to country n from the total emissions.

Our model adaptations

Addapt:

- a_n action of player $n \in \mathcal{N} := \{1, \ldots, N\}$,
  $\Rightarrow a_n \in [0, e_n^{max}] = \mathcal{A}_n$, $\mathcal{A} = \prod_{n \in \mathcal{N}} \mathcal{A}_n$

- $\theta^{\mathrm{AT}} : \mathcal{X} \times \mathcal{A}$: Atmospheric temperature from climate model.
- $x = (C, \theta) \in \mathcal{X}$ the state, describing the climate

- Payoff function
$$u_n(x, a) = B_n(a_n) - w_n D\left(\theta^{\mathrm{AT}}(x, a)\right)$$
- D **is now global!** (i.e. not D_n anymore)
- $w_n \geq 0$ multiplier of damage to n, $D_n \simeq w_n D$[1]

[1] [Tulkens, 2019]

About the Model

New Assumptions:

- On B_n and D:

$$B_n(a_n) : \text{Increasing, quasi-Concave} \quad (B_n' > 0, B_n'' \leq 0)$$
$$D\left(\theta_t^{\mathrm{AT}}\right) : \text{Increasing, Convex} \quad (D_n' > 0, D_n'' \geq 0)$$

Interests:

- With a climate model
- Simulation possible
- Simpler but is **what literature do without noticing!!**[2]

[2] [Tulkens, 2019]

Complete Formalism of Our Game

Let us include time dependence:

Definition (Planning game)

For a given time $T \in \mathbb{N}$, for any $x_1 \in \mathcal{X}$, $(a_n^T, a_{-n}^T) \in \mathcal{A}^T$, define player n's utility function $U_n^T : \mathcal{X} \times \mathcal{A}^T \to \mathbb{R}$ as

$$U_n^T(x_1, a_n^T, a_{-n}^T) = \sum_{t=1}^{T} u_n(x_t, a_{nt}, a_{-nt}).$$

The instantaneous utility is given by a benefit function $B_{nt} : \mathcal{A}_{nt} \to \mathbb{R}$ minus a weighted damage $D : \mathbb{R} \to \mathbb{R}$, for some weights $w_n > 0$, $\forall n \in \mathcal{N}$ as follows

$$u_n(x_t, a_{nt}, a_{-nt}) = B_{nt}(a_{nt}) - w_n D\left(\theta_{t+1}^{\mathrm{AT}}\right).$$

Potential Game?

Introducing some particular games:

Definition (Weighted Potential Game)

Game $(\mathcal{N}, \mathcal{A}, (u_n)_{n\in\mathcal{N}})$ is a weighted potential game i.f.f. there exists a function $\phi : \mathcal{A} \longrightarrow \mathbb{R}$ and (ω_n) positive weights, s.t, for all $n \in \mathcal{N}$, $a_n, \tilde{a}_n \in \mathcal{A}_n$ and $a_{-n} \in \mathcal{A}_{-n}$,

$$u_n(a) - u_n(\tilde{a}_n, a_{-n}) = \omega_n \left[\phi(a) - \phi(\tilde{a}_n, a_{-n})\right].$$

Is our game potential?

Our one-stage game is! $\phi(x, a) = \sum_{n=1}^{N} \frac{B_n(a_n)}{w_n} - D\left(\theta^{\mathrm{AT}}(x, a)\right)$.
But what about the planning game?

State transitive Potential game

New kind of potential game

Definition (State transitive Potential Game)

A game is state transitive weighted potential if there exists $\phi : \mathcal{X} \times \mathcal{A} \to \mathbb{R}$ and positive weights $(\omega_n)_{n \in \mathcal{N}}$ such that the following holds for any $a \in \mathcal{A}$, $\forall x, \tilde{x} \in \mathcal{X}$,

$$u_n(x, a) - u_n(\tilde{x}, a) = \omega_n \left(\phi(x, a) - \phi(\tilde{x}, a)\right), \quad \forall n \in \mathcal{N}.$$

Our one-stage game is state transitive!
Then our Planning game is too with following potential:

$$\Phi^T(x_1, a^T) = \sum_{t=1}^{T} \phi(x_t, a_t).$$

Potential make finding NE easier

Local maximum of potential ϕ are NE

Proposition (Existence of NE)

If the potential ϕ admits at least one maximum over the set $\mathcal{A}$, then the game Γ has at least one pure Nash equilibrium.

Reasonable condition

Proposition (Existence in our game)

Assume that functions B_{nt} and $D \circ \theta^{\mathrm{AT}}$ are continuous with respect to $a \in \mathcal{A}_t$ and $x \in \mathcal{X}$ for all $n \in \mathcal{N}$ and $t \in \mathcal{T}$.
Then there exists at least one pure Nash equilibrium in our game.

Remark: Pure NE mean there is no randomness in strategy definition.

First Order Condition (FOC) for the NE

Theorem (FOC for the Planning game)

Let a^{T*} be a pure Nash equilibrium of the game Γ^T. There exist positive $(\underline{\lambda}_{nt})$ and $(\overline{\lambda}_{nt})$ for all $t \in \mathcal{T}$ such that a^{T*} must verify the system of $N \times T$ equations

$$B'_{nt}(a^*_{nt}) = w_n \sum_{s=t}^{T} \sum_{r=s}^{T} \frac{D'(\theta^{\mathrm{AT}*}_{r+1}) d_\theta^\top \mathbf{A}_\theta^{r-s} b_\theta}{C^{\mathrm{AT}*}_{s+1}} d_C^\top \mathbf{A}_C^{s-t} b_C + \overline{\lambda}_{nt} - \underline{\lambda}_{nt},$$

with the dynamic of $(\theta^{\mathrm{AT}*}_{t+1})$ and $(C^{\mathrm{AT}*}_{t+1})$ given by the dynamic with a_n^{T*} and the KKT admissible conditions,

$$\underline{\lambda}_{nt}(e^{\min}_{nt} - a^*_{nt}) = 0, \overline{\lambda}_{nt}(a^*_{nt} - e^{\max}_{nt}) = 0,$$
$$\underline{\lambda}_{nt}, \overline{\lambda}_{nt} \geq 0, \forall n \in \mathcal{N}, \forall t \in \mathcal{T}.$$

What can we say about that?

Proposition (Uniqueness Conditions)

Assuming as before. $\mathbf{C}^{\mathrm{AT}}(x)$, $\Theta^{\mathrm{AT}}(x)$ *the possible state from* x.

- $\forall C^{\mathrm{AT}}, \tilde{C}^{\mathrm{AT}} \in \mathbf{C}^{\mathrm{AT}}(x)$, *such that* $C^{\mathrm{AT}} < \tilde{C}^{\mathrm{AT}}$, *we have*

$$\frac{D'(\tilde{\theta}^{\mathrm{AT}})}{D'(\theta^{\mathrm{AT}})} \geq \frac{\tilde{C}^{\mathrm{AT}}}{C^{\mathrm{AT}}}, \qquad \textbf{\textit{(Damage fast exp-like)}}.$$

- $\exists n \in \mathcal{N}$ *such that* $\forall a_n, \tilde{a}_n \in \mathcal{A}_n$, $a_n < \tilde{a}_n$, *we have*

$$\frac{B_n'(\tilde{a}_n)}{B_n'(a_n)} \leq \frac{C^{\mathrm{AT}}}{\tilde{C}^{\mathrm{AT}}}, \qquad \textbf{\textit{(Benefits slow log-like)}}.$$

- D *is twice differentiable and for* $x \in \mathcal{X}$, $\forall \theta^{\mathrm{AT}} \in \Theta^{\mathrm{AT}}(x)$

$$D''(\theta^{\mathrm{AT}}) \geq D'(\theta^{\mathrm{AT}})/\tilde{b}_\theta, \qquad \textbf{\textit{(Damage fast exp-like)}}.$$

Why such condition are important?

These conditions are not present in the literature.
They incentivise to reduce emissions.

Assumption (Benefits on same scale than damage)

For all $n \in \mathcal{N}$, $t \in \mathcal{T}$, $B'_{nt}(e^{\max}_{nt}) = 0$ or be negligible compared to the damages induced by players' emissions.

Literature, only quadratic damages:

- $D(X) = 1.145X^2$,
- $D(X) = 0.267X^2$,
- $D(X) = 1.1X^2 - 2.46X$,

Quadratic is too weak! It does not satisfy the condition on the damage all!

Outline

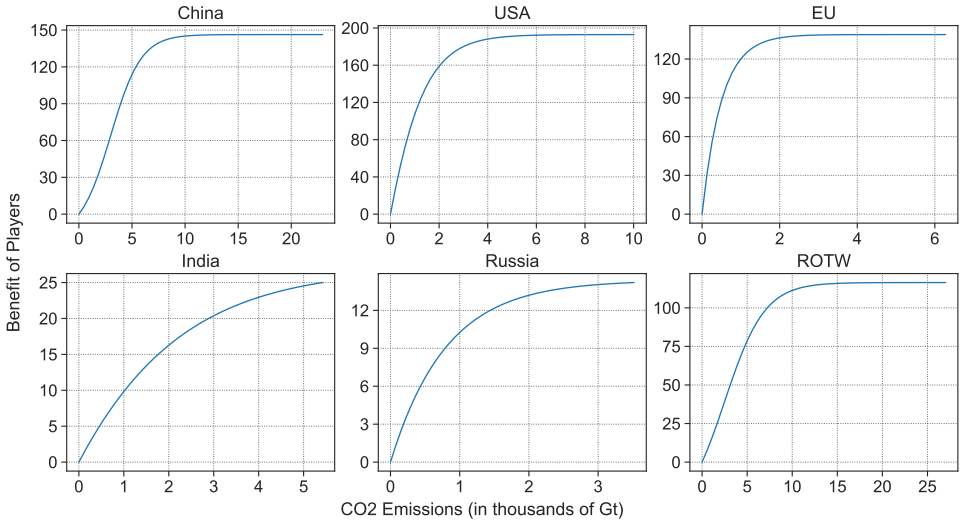
1. Models

2. General Game

3. Numerical Study
   - Methodology
   - Efficiency of Nash equilibrium
   - Comparison to Social Optimum (SO)
   - Same benefits different actions

Players' characteristics

Benefit function: in (10^{11} US$ 2011)



How the game is played

We need to compare efficiency in 2100:
$t_0 = 2020$, $t_{\max} = 2100$, $\Delta t = 5 \Rightarrow T = 16$

Planning game: 1 game with $T = 16$

Repeated One-stage: 16 games with $T = 1$:
$\Rightarrow$ at t the game is played for **1** step with state $x_t = (C_t, \theta_t)$
$\Rightarrow$ the actions provide a $x_{t+1} = (C_{t+1}, \theta_{t+1})$

Receding horizon: 16 games with $T = 16$
$\Rightarrow$ at t the game is played for **16** step with state $x_t = (C_t, \theta_t)$
$\Rightarrow$ the actions provide a $x_{t+1} = (C_{t+1}, \theta_{t+1})$

Outcomes change a lot depending on the ways you play the game

Impact of the method of planning

We will look at:

- Efficiency of different ways the game is played:
  $\Rightarrow$ temperature increase and actions
- Comparing Nash equilibrium to Social Optimum
  $\Rightarrow$ temperature, actions and utility
- Efficiency of different solution concepts in utility space for 2 players

One-Stage game

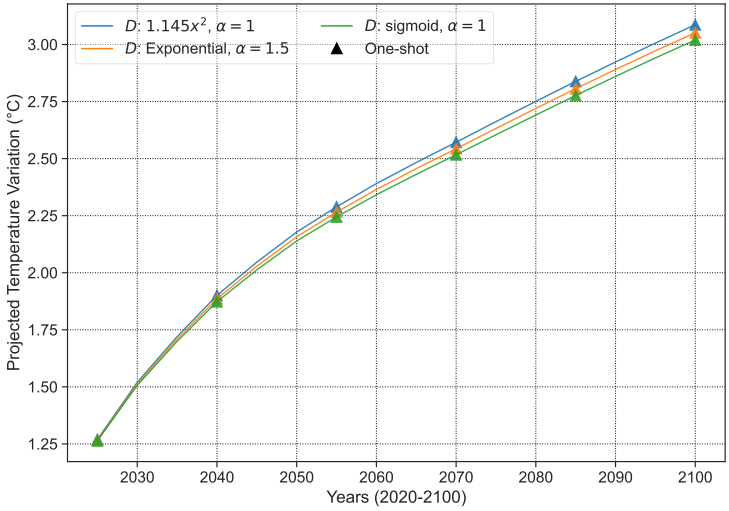


Figure: Temperature Trajectory from 2020 to 2100 (C°)

One-Stage and Planning games

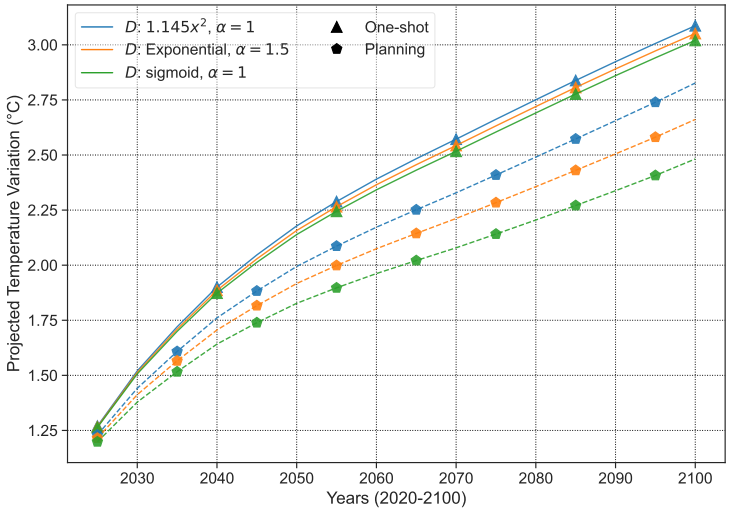


Figure: Temperature Trajectory from 2020 to 2100 (C°)

All games Temperature

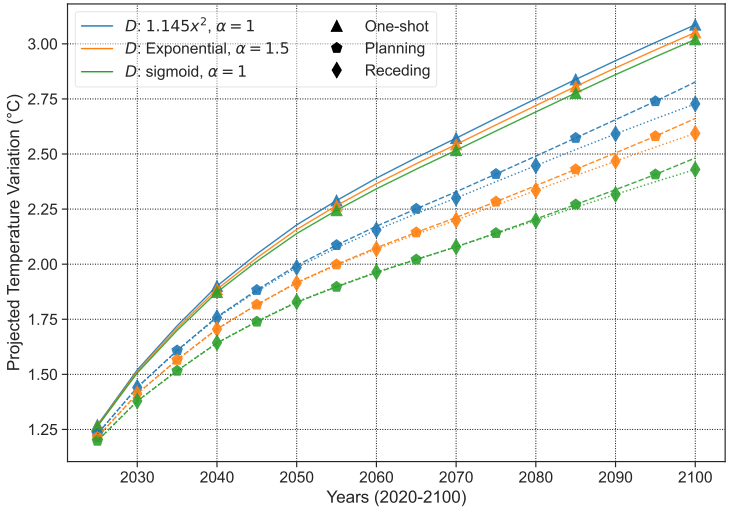


Figure: Temperature Trajectory from 2020 to 2100 (C°)

All game Actions

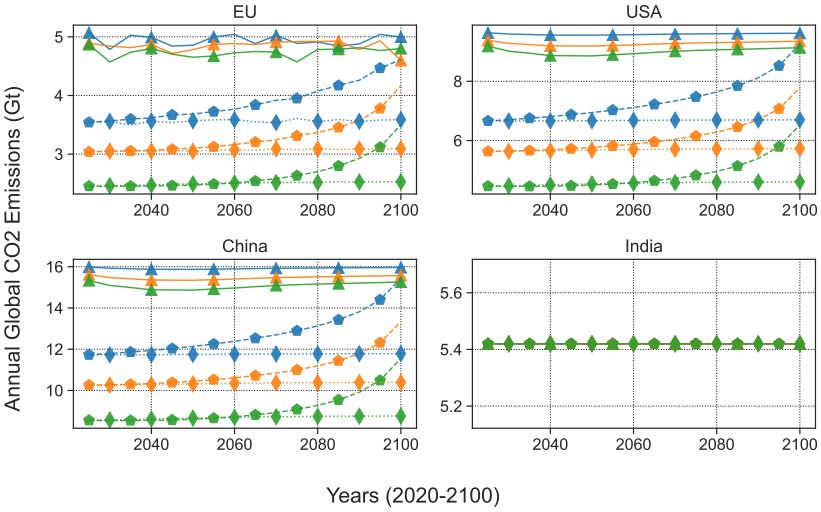


Figure: Actions of EU, USA, China and India (GtCO2).

What can we say?

We observe[3]:

- The strongest the damage function the lower the temperature.
- We always have θ^{AT}: One-stage $>$ Planning $>$ Receding.
- We always have Emissions: One-stage $>$ Planning $>$ Receding.
- Action behaviors per game
  - In One-stage action almost does not change
  - In Planning, it increases at the end (do not consider period after 2100)
  - In Receding, it stagnates but much lower
- There is players (like India) that never change their actions: e_{max} is dominating strategy.

Intuitive results are verified in our model.

[3] Under constant action set / benefit function / $\mathrm{GDP}^{\mathsf{max}}$

Comparison to utilitarian Social Optimum (SO)

What is utilitarian Social Optimum?

The social optimum is the configuration of actions that maximize the sum of all players' utilities.
$\Rightarrow$ Assess how much selfish behaviors may induce a loss of optimality.

Definition (Social Optimum problem)

The social optimum problem, starting from a initial climate state $x \in \mathcal{X}$, can be mathematically represented as:

$$\max_{a^T \in \mathcal{A}^T} \bar{U}^T(x, a^T) := \sum_{n=1}^{N} U_n^T(x, a_n^T, a_{-n}^T)$$

In the following, only look at One-shot and Receding

NE versus SO: temperature

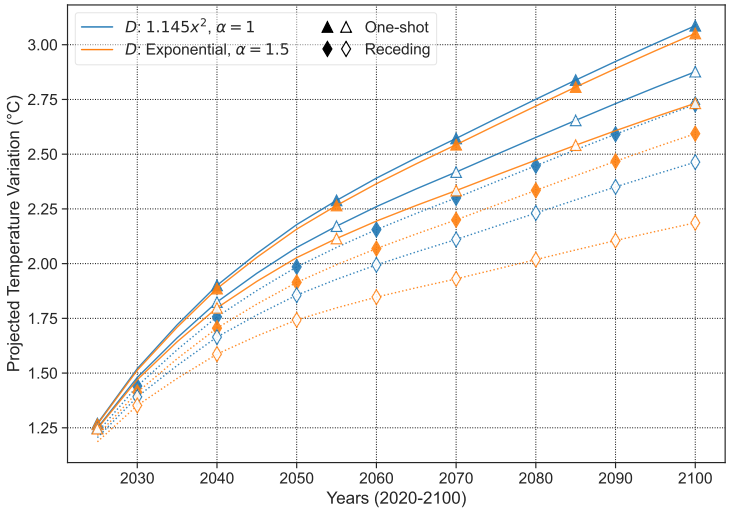


Figure: θ^{AT}: Nash Equilibrium $>$ Social optimum

NE versus SO: utilities

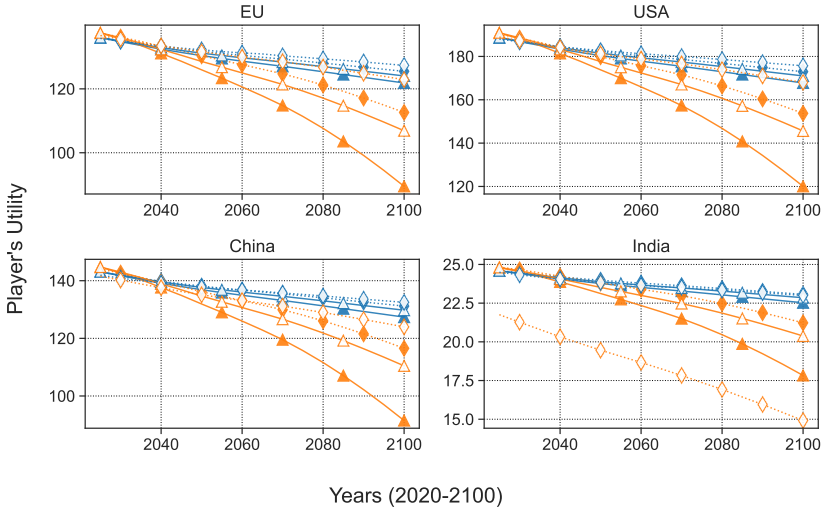


Figure: Utility - better with cooperation for almost all player.

What can we say?

We observe[4]:

- We always have θ^{AT}: Nash Equilibrium $>$ Social optimum.
- We always have Emissions: Nash Equilibrium $>$ Social optimum.
- Utility are better with cooperation for almost all player
  $\Rightarrow$ for One-shot and Receding game
- At the end of century there is a big difference of utility between NE and SO
- **Receding SO isn't interesting solution compare to NE for India** $\Rightarrow$ it cannot be a Nash equilibrium
- There is players (like India) that never change their actions: e_{max} is dominating strategy except in social optimum.

Cooperation: not always profitable on short term for CO2 based economies.

[4] Under constant action set / benefit function / $\mathrm{GDP}^{\mathsf{max}}$

Efficiency Solution concept for two players

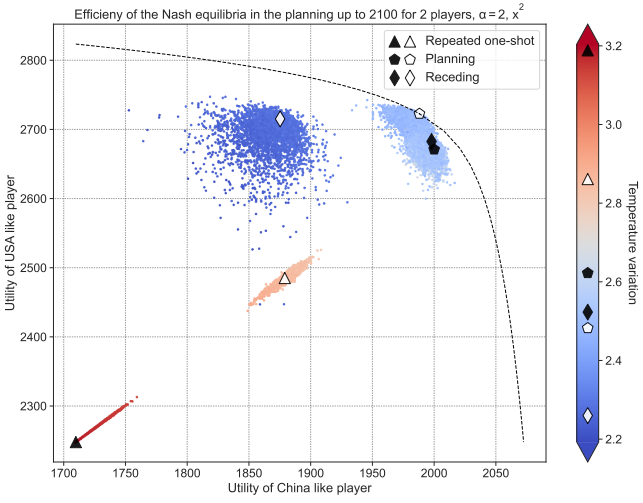


Figure: Aggregated USA and China. Efficiency of solution concepts individually.

Conclusion

- **Contributions**
  - New structured game including climate model
  - Equilibrium analysis: existence and explicit characterization in quadratic case
  - Important conditions
    - for uniqueness of NE that damage functions must to satisfy
    - under which NE = SO (one-shot and planning)
  - Approach that work with any climate model
  - Simulator: allowing to find equilibrium to see behaviors of players and impacts on climate, available on Github (Beta) [Couthures, 2023]

- **Future research topics**
  - Opinion dynamic: to connect player with economic relation
  - Include mediator: United Nation with goal to reach target temperature
  - Better economy modeling: dynamical system with dynamic action sets
  - Social Optimum: Look at other fairness metrics than utilitarian

To take away

Key ideas:

- Our game is potential for any Climate Model
- Existence of Nash Equilibria for very weak conditions
- Uniqueness of Nash Equilibrium is related on the structure of Benefit versus Damage
- Quadratic damage function are too weak to change the behaviors of players in simulation

About the Literature Model

Assumptions:

- $B_n(0) = 0$ and $D_n(0) = 0$: No benefit or damage without emissions.
- Benefits[5] and damages[6]:

$$B_n(a_n) : \text{Increasing, Concave} \quad (B_n' > 0, B_n'' \leq 0)$$
$$D_n\left(\sum a_k\right) : \text{Increasing, Convex} \quad (D_n' > 0, D_n'' \geq 0)$$

Issues:

- Without a climate model
- No simulation possible that capture climate change
- No equilibrium uniqueness guarantee nor determination

[5] [Finus, 2002]
[6] [Weitzman, 2010]

Fixed parameters

- $N = 6$
- Characteristics of players

Player	e_n^{max} (GtCO2)	$\mathrm{GDP}_n^{\max}$ (10^{11} US$ 2011)	w_n
China	11	146.3	1.1847
USA	7	192.9	1.1941
EU	4	138.9	1.1248
India	3	25	0.9074
Russia	2	14.2	1.2866
ROTW	5	116.4	1.1847

- Climate model:
  Carbon Cycle [Joos et al., 2013] + Temperature Dynamic [Geoffroy et al., 2013] (best fit IPCC prediction)

Discretized “Variable”

Candidates damage function : in (% $\mathrm{GDP}^{\max}$)

